Optimal Pilot-to-Data Power Ratio for Diversity Combining with Imperfect Channel Estimation

Yong Peng, Shengshan Cui, and Roy You

***Abstract*— The bit error probability for BPSK modulation over correlated Rice fading channel is derived for minimum mean square error channel estimation using pilot symbol assisted modulation and diversity combining at the receiver. The optimal pilot-to-data power ratio is studied for different Rice K factors. The theoretical result is verified by Monte-Carlo simulation with space diversity fading model.**

***Index Terms*— Diversity combining, imperfect channel estimation, correlated fading, Rice fading.**

I. Introduction

DIVERSITY combining has often been used to combat the deleterious effect of channel fading where the channel state varies randomly over time, space and frequency [1]. When the channel state information (CSI) is known perfectly at the receiver, various diversity combining techniques can be used to improve the bit error rate (BER) of the system [2]. However, in real practice, the CSI available at the receiver is usually imperfect, thus degrading the combining performance. In [3], the effect of imperfect channel estimation on BPSK modulation with maximal ratio combining (MRC) is investigated. In [4], the optimal combining for BPSK modulation over correlated Rayleigh fading channels with minimum mean square error (MMSE) channel estimation is derived and the BER performance is presented in analytical form, which generalizes the special cases of diversity combining with imperfect channel estimation for uncorrelated diversity branches in [5] and diversity combining for correlated branches but with perfect channel estimation in [6]. In this letter, we further apply the framework developed in [4] to investigate the BER performance of diversity combining of BPSK modulation with imperfect CSI over correlated Rice fading channel, where line of sight (LoS) paths exist between the transmitter and the receiver. Furthermore, for channel estimation using pilot symbol assisted modulation (PSAM) scheme [7], we study the pilot-to-data power ratio to optimize the BER performance under fixed total transmitting power.

This letter is organized as follows. In Section I, the system model is introduced and the optimal combining technique under Rice fading channel is derived. In Section III, the analytical BER expressions are given for both perfect and imperfect CSI. The optimal pilot-to-data power ratio is studied in Section IV. Section V presents the numerical results using a space diversity fading model. Section VI concludes the letter.

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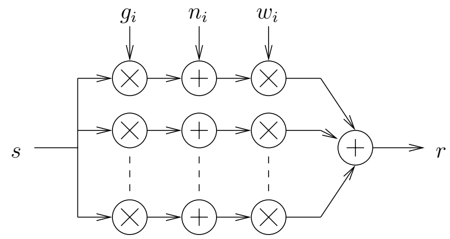

Fig. 1. A general single user diversity combiner.

II. Optimal Diversity Combining with MMSE

A diversity combining system is shown in Fig.1, where a BPSK symbol $s \in \{\sqrt{E_s}, -\sqrt{E_s}\}$ with equal probability is transmitted over a correlated Rice fading channel with N diversity branches. The additive white Gaussian noise $\mathbf{n} = [n_1, n_2, \ldots, n_N]^H$ is assumed to be proper complex [8] with $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, N_0 I_N)$ where $[\cdot]^H$ denotes the Hermitian operator and I_N denotes the identity matrix. The channel state $\mathbf{g} = [g_1, g_2, \ldots, g_N]^H$ is also assumed to be proper complex Gaussian, with $\mathbf{g} \sim \mathcal{CN}(\mathbf{u}, C_{\mathbf{gg}})$. $\mathbf{u} = [u_1, u_2, \ldots, u_N]^H$ is the LoS component. The channel covariance matrix $C_{\mathbf{gg}} = \mathbf{U}\Lambda_{\mathbf{gg}}\mathbf{U}^H$ can be diagonalized by the unitary matrix $\mathbf{U}$ with eigenvalue matrix $\Lambda_{\mathbf{gg}} = \mathrm{diag}(\sigma^2_{\bar{g}_1}, \sigma^2_{\bar{g}_2}, \ldots, \sigma^2_{\bar{g}_N})$. Without loss of generality, we assume that $\mathbf{u}^H\mathbf{u} + \mathrm{tr}(C_{\mathbf{gg}}) = 1$, where $\mathrm{tr}(\cdot)$ denotes the trace of a matrix. The Rice K factor is defined as the ratio between the total LoS and total diffuse power

$$K \triangleq \frac{\mathbf{u}^H\mathbf{u}}{\mathrm{tr}(C_{\mathbf{gg}})}. \tag{1}$$

We further assume that the mean channel gain is the same for each diversity branch ($|u_n|^2 = (K/N)\sum_{i=1}^{N} \sigma^2_{\bar{g}_i}$ for all n). The channel is also assumed to be block fading with block length $L = \lfloor 1/2 f_D T_s \rfloor$, where f_D is the Doppler frequency spread and T_s is the symbol duration.

Using PSAM, M pilot symbols with amplitude $\sqrt{E_p}$ are inserted in each fading block for channel estimation. We define $\gamma_b \triangleq \frac{E_b}{N_0}$ as the effective SNR per bit, where

$$E_b \triangleq \frac{(L-M)E_s + ME_p}{L-M}. \tag{2}$$

The normalized channel observation is

$$\mathbf{h} = \mathbf{g} + \frac{\mathbf{n}}{\sqrt{ME_p}}, \tag{3}$$

where $\mathbf{h} \sim \mathcal{CN}(\mathbf{u}, C_{\mathbf{hh}})$. $C_{\mathbf{hh}} = \mathbf{U}(\Lambda_{\mathbf{gg}} + (N_0/ME_p)I_N)\mathbf{U}^H$ can also be diagonalized by unitary matrix $\mathbf{U}$. Using MMSE, the estimation of the channel state is

$$\mathbf{m} = \mathbf{u} + C_{\mathbf{gh}} C_{\mathbf{hh}}^{-1}(\mathbf{h} - \mathbf{u}). \tag{4}$$

$C_{\mathbf{gh}} = E[(\mathbf{g} - \mathbf{u})(\mathbf{h} - \mathbf{u})^H] = \mathbf{U}\Lambda_{\mathbf{gh}}\mathbf{U}^H$ is the cross-covariance matrix and $\mathbf{m} \sim \mathcal{CN}(\mathbf{u}, C_{\mathbf{mm}})$, where $C_{\mathbf{mm}} = C_{\mathbf{gh}} C_{\mathbf{hh}}^{-1} C_{\mathbf{gh}}^H = \mathbf{U}\Lambda_{\mathbf{mm}}\mathbf{U}^H$. The channel estimation error $\mathbf{e} = \mathbf{g} - \mathbf{m}$ also has Gaussian distribution with zero-mean and covariance matrix of

$$C_{\mathbf{ee}} = C_{\mathbf{gg}} - C_{\mathbf{mm}} = \mathbf{U}(\Lambda_{\mathbf{gg}} - \Lambda_{\mathbf{mm}})\mathbf{U}^H. \tag{5}$$

Applying diversity combining by multiplying each diversity branch with weight coefficients $\mathbf{w} = [w_1, w_2, \ldots, w_N]^H$, the combiner output is

$$r = \mathbf{w}^H(\mathbf{g}s + \mathbf{n}). \tag{6}$$

With maximum likelihood detection, the decision rule is

$$|r - \mathbf{w}^H\mathbf{m}s_0|^2 \underset{\hat{s}=s_1}{\overset{\hat{s}=s_0}{\lessgtr}} |r - \mathbf{w}^H\mathbf{m}s_1|^2. \tag{7}$$

Following similar steps as in [4], the conditional error probability can be derived as

$$\Pr(e|s, \mathbf{h}) = Q\left(\sqrt{\frac{2|\mathbf{w}^H\mathbf{m}|^2 E_s}{(\mathbf{w}^H(C_{\mathbf{ee}}E_s + N_0 I_N)\mathbf{w})}}\right), \tag{8}$$

where the optimal weighting coefficients is

$$\mathbf{w} = (C_{\mathbf{ee}}E_s + N_0 I_N)^{-1}\mathbf{m}. \tag{9}$$

The corresponding BER is

$$P_e = \int Q(\sqrt{2\mathbf{m}^H(C_{\mathbf{ee}}E_s + N_0 I_N)^{-1}\mathbf{m}E_s}\ )\ p(\mathbf{m})\ d\mathbf{m}. \tag{10}$$

III. BER Evaluation

To evaluate the BER, we use the approach in [2] with the alternative expression for the $Q(\cdot)$ function

$$Q(x) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \exp\left(-\frac{x^2}{2\sin^2\theta}\right) d\theta. \tag{11}$$

A. Evaluation for Perfect Channel Estimation

When the CSI is perfectly known, $\mathbf{m} = \mathbf{h} = \mathbf{g}$. Using Eqs.(10) and (11), P_e can be written as

$$\begin{aligned} P_e &= \frac{1}{\pi}\int_{\mathbf{g}}\int_0^{\pi/2} \exp\left[-\frac{\mathbf{g}^H(N_0 I_N)^{-1}\mathbf{g}E_s}{\sin^2\theta}\right]\frac{1}{\pi^N \det(C_{\mathbf{gg}})} \\ &\quad \cdot \exp\left[-(\mathbf{g}-\mathbf{u})^H C_{\mathbf{gg}}^{-1}(\mathbf{g}-\mathbf{u})\right] d\theta\ d\mathbf{g} \\ &= \frac{1}{\pi}\int_0^{\pi/2}\left\{\det\left[\frac{(E_s/N_0)C_{\mathbf{gg}}}{\sin^2\theta} + I_N\right]\right\}^{-1} \\ &\quad \cdot \exp\{-\mathbf{u}^H[C_{\mathbf{gg}} + (E_s/N_0)^{-1}\sin^2\theta I_N]^{-1}\mathbf{u}\}\ d\theta. \end{aligned} \tag{12}$$

In Rayleigh fading where $\mathbf{u} = \mathbf{0}$, the BER has a close-form solution [3]

$$P_e = \frac{1}{2}\sum_{n=1}^{N}\left(\prod_{\substack{i=1\\ i\neq n}}^{N}\left(1 - \frac{\lambda_i'}{\lambda_n'}\right)\right)^{-1}\left[1 - \sqrt{\frac{\lambda_n'}{1+\lambda_n'}}\right] \tag{13}$$

where

$$\lambda_n' \triangleq \frac{\sigma_{\tilde{g}_n}^2 E_s}{N_0}. \tag{14}$$

In Rice fading where $\mathbf{u} \neq \mathbf{0}$, P_e can not be simplified any further. However, Since (12) is a finite-range integral, the BER can easily be evaluated numerically for any given $\mathbf{u}$.

B. Evaluation for Imperfect Channel Estimation

With imperfect channel estimation, using Eqs.(10) and (11), the BER can be expressed as

$$\begin{aligned} P_e &= \frac{1}{\pi}\int_{\mathbf{m}}\int_0^{\pi/2} \exp\left[-\frac{\mathbf{m}^H\Sigma^{-1}\mathbf{m}E_s}{\sin^2\theta}\right]\frac{1}{\pi^N \det(C_{\mathbf{mm}})} \\ &\quad \cdot \exp\left[-(\mathbf{m}-\mathbf{u})^H C_{\mathbf{mm}}^{-1}(\mathbf{m}-\mathbf{u})\right] d\theta\ d\mathbf{m} \\ &= \frac{1}{\pi}\int_0^{\pi/2}\left\{\det\left[\frac{C_{\mathbf{mm}}\Sigma^{-1}E_s}{\sin^2\theta} + I_N\right]\right\}^{-1} \\ &\quad \cdot \exp\left\{-\mathbf{u}^H[C_{\mathbf{mm}} + \Sigma E_s^{-1}\sin^2\theta]^{-1}\mathbf{u}\right\} d\theta. \end{aligned} \tag{15}$$

where $\Sigma = C_{\mathbf{ee}}E_s + N_0 I_N$. The closed form expression of BER in Rayleigh correlated fading is given in [4]

$$P_e = \frac{1}{2}\sum_{n=1}^{N}\left(\prod_{\substack{i=1\\ i\neq n}}^{N}\left(1 - \frac{\lambda_i}{\lambda_n}\right)\right)^{-1}\left[1 - \sqrt{\frac{\lambda_n}{1+\lambda_n}}\right] \tag{16}$$

where

$$\lambda_n \triangleq \frac{\sigma_{\tilde{g}_n}^2 E_s}{N_0}\cdot\frac{ME_p}{E_s + ME_p + N_0/\sigma_{\tilde{g}_n}^2}. \tag{17}$$

Again, (15) must be evaluated by numerical integration when the channel is Rice correlated fading with $\mathbf{u} \neq \mathbf{0}$.

IV. Pilot-to-Data Power Ratio Optimization

For pilot symbol assisted modulation, a trade-off exists for the pilot and data symbol power. While more pilot power leads to better channel estimation, the overhead imposed by pilot symbol reduces the effective SNR γ_b. In this section, we study the power ratio between pilot and data symbol power for optimal BER performance.

We define the ratio between the pilot power and the total transmitted power as

$$\beta \triangleq \frac{ME_p}{ME_p + (L-M)E_s}, \quad \beta \in (0, 1). \tag{18}$$

For Rayleigh fading case, substituting E_b and β into (17) and assuming high SNR ($\gamma_b \gg 1$), we have

$$\begin{aligned} \lambda_n &= \frac{\sigma_{\tilde{g}_n}^4 (L-M)\beta(1-\beta)\gamma_b^2}{\sigma_{\tilde{g}_n}^2[\beta(L-M) + (1-\beta)]\gamma_b + 1} \\ &\approx \frac{(L-M)\beta(1-\beta)}{(L-M)\beta + (1-\beta)}\cdot\sigma_{\tilde{g}_n}^2\cdot\gamma_b. \end{aligned} \tag{19}$$

Since P_e decreases monotonically with increasing λ_n, lower BER can be obtained by maximizing $\frac{(L-M)\beta(1-\beta)}{(L-M)\beta+(1-\beta)}$. The optimal β value is approximately

$$\beta \approx \frac{\sqrt{L-M} - 1}{(L-M) - 1}. \tag{20}$$

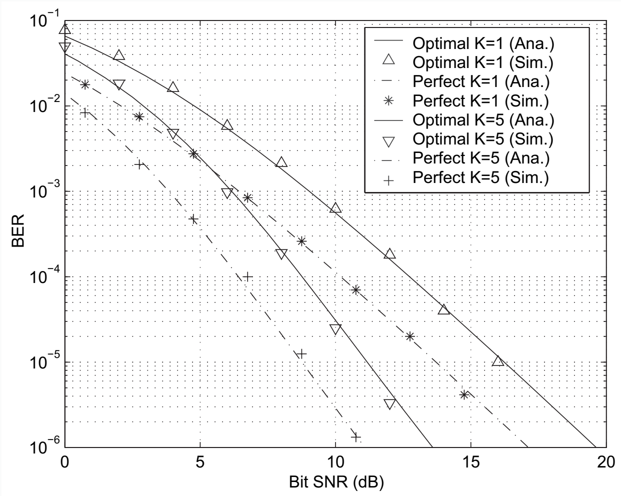

Fig. 2. Comparison of the analytical and simulation results of BER P_e vs. SNR γ_b with different Rice K factors using spatial diversity model ($N = 3$, $d = .15$m, $f_D T_s = .05$, $L = 10$, $M = 1$, $E_p = 3E_s$).

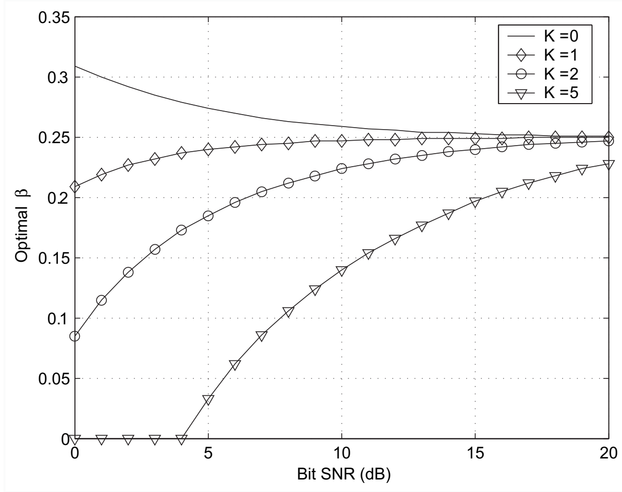

Fig. 3. The optimum value of β vs. SNR γ_b with Rice factors $K = 0, 1, 2, 5$ ($N = 3$, $d = .15$m, $f_D T_s = .05$, $L = 10$, $M = 1$).

Substituting (18) into (20), we get

$$\frac{ME_p}{(L-M)E_s} \approx \frac{1}{\sqrt{L-M}}. \tag{21}$$

With typical $f_D T_s$ parameters, e.g. $f_D T_s = .05$, $L = 10$. If we want to use $E_p = E_s$, we should have $M \approx 3$. If we want to use $M = 1$, then we should have $E_p \approx 3E_s$.

For Rice fading case, since there is no closed form expression for P_e, the optimal β can only be obtained by numerical simulation, which is demonstrated in next section.

V. Numerical Results

For simulation, a spatial diversity fading model is used [9]

$$C_{\mathbf{gg}}(m,n) = \sigma_g^2 J_0 \left(2\pi \frac{|m-n|\, d \cdot f_c}{c} \right), \tag{22}$$

where d is the distance between two adjacent antennas, f_c is the carrier frequency, and c is the speed of light. $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind. In our case, three antennas ($N = 3$) are spaced inline equally with separation distance $d = .15$m. We also assume that $f_D T_s = .05$, which implies $L = 10$. One pilot symbol per fading block ($M = 1$) is used and the pilot-to-data power ratio is set to $E_p = 3E_s$, which gives the optimal $\beta = .25$ for Rayleigh fading at high SNR. Fig. 2 shows both the analytical and the Monte-Carlo simulation results of the BER performance for the optimal diversity combining using PSAM and the maximal ratio combining with perfect channel knowledge for $K = 1$ and $K = 5$. It can be seen that the analytical results agree with the Monte-Carlo simulations quite well. As expected, the BER decreases with larger Rice K factor since there is stronger LoS component and less channel uncertainty.

The trade-off between pilot and data symbol power is investigated numerically with respect to Rice K factor and effective SNR γ_b. The result is plotted in Fig. 3. With large Rice K factor, since there is less channel uncertainty, less pilot symbol energy is needed for channel estimation. The optimal β value is also dependent on the effective SNR γ_b. At high SNR, the channel estimation error dominates. Therefore, the optimal proportion should be used to achieve the balance between estimation overhead and accuracy. At low SNR, the BER performance is dominated by channel noise and the channel estimation can not be significantly improved even if more power is allocated. Therefore, if a reliable LoS component exists, less power is needed on channel estimation.

VI. Conclusion

In this letter, we have shown that the diversity combining performance over Rice correlated fading channel is dependent on the Rice K factor as well as the power ratio between the pilot and data symbols. When more signal propagate through the LoS path between transmitter and receiver, there is less channel uncertainty and less resource is needed for channel estimation.

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