

Analysis of UWB Transmitted-Reference Communication Systems in Dense Multipath Channels

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Abstract—Transmitted-reference (TR) signaling, in conjunction with an autocorrelation receiver (AcR), offers a low-complexity alternative to Rake reception. Due to its simplicity, there is renewed interest in TR signaling for ultrawide bandwidth (UWB) systems. To assess the performance of these systems, we develop an analytical framework based on the sampling expansion approach. In particular, we derive closed-form expression for the bit-error probability (BEP) of TR signaling with AcR that can be used to exploit multipath diversity inherent in wideband channels. We further extend our analysis to the BEP derivation of modified AcR with noise averaging. Our methodology does not require the Gaussian approximation and is applicable for any fading scenario, provided that the correlator output signal-to-noise ratio (SNR) can be characterized in terms of a characteristic function. We show that the validity of the conventional Gaussian approximation depends on the time-bandwidth product and the number of transmitted pulses per symbol. Our results enable the derivation of a computationally simple lower bound on the BEP of TR signaling with AcR. This lower bound allows us to obtain the SNR penalty associated with an AcR, as compared with *All-Rake* and *Partial-Rake* receivers.

Index Terms—All-rake (ARake) receiver, autocorrelation receiver (AcR), partial-rake (PRake) receiver, sampling expansion, transmitted-reference (TR).

I. INTRODUCTION

RECENTLY, there has been an increasing interest in ultrawide bandwidth (UWB) systems for future military, homeland security, and commercial applications [1]–[6]. The key motivations for using these systems are the ability of UWB signals to finely resolve multipath components, as well as the availability of technology to implement and generate UWB signals with relatively low complexity. Fine delay resolution properties make UWB radio a viable candidate for communications in dense multipath environments, such as short-range or indoor wireless communications [7]–[10].

UWB systems involve the transmission of a train of extremely short pulses by employing either a time-hopping (TH) or a direct

sequence (DS) technique to achieve multiple access, and either pulse position modulation (PPM) or pulse amplitude modulation (PAM) for data transmission [1], [2], [11]. In addition, a combination of these modulations is also possible [12]. To demodulate UWB signals, the reference signal can be locally generated by the receiver, which requires accurate acquisition and channel estimation. We refer to this class of systems as locally generated reference (LGR) systems.

As an alternative to LGR systems, a reference signal can be transmitted along with the data. Such a signaling scheme, referred to as transmitted-reference (TR) signaling, was first considered in the 1950s [13]–[15], when it was proposed for military spread-spectrum communications [16]. It involves the transmission of a reference and data signal pair, separated either in time [14], [15] or in frequency [17]. In order for this pair of separated signals to experience the same channel, either the time separation must be less than the coherence time, or the frequency separation must be less than the coherence bandwidth. The receiver can simply be an autocorrelation receiver (AcR). The bit-error probability (BEP) of TR signaling with AcR was derived in [18] and [19] by representing the output of the AcR as a Hermitian quadratic form in complex normal variates [20]. The probability distribution of the output of an analog cross correlator with noisy reference inputs was derived for low-pass signals in [21] and for bandpass signals in [22]–[24].

Due to its simplicity, there is renewed interest in TR signaling for UWB systems [25]–[28] since conventional Rake reception in these systems requires a large number of Rake fingers to capture most of the multipath energy [8]–[10]. Moreover, channel estimation error can deteriorate the performance of Rake receivers. A typical approach to analyzing the BEP performance of TR signaling with AcR in UWB systems is to obtain the conditional BEP, using a Gaussian approximation of the noise components at the output of the AcR [25]–[28]. The BEP of TR signaling can then be obtained by numerically averaging the conditional BEP with a quasi-analytical/experimental approach [26], [27] or a quasi-analytical/simulation approach [28]. Although one can also use the derived BEP in [18] and [19], these results are only applicable to Rayleigh fading. Since the amplitude distribution of the resolved multipaths in typical UWB channels can be drastically different from the Rayleigh distribution [9], [10], there is a compelling need for analytical expressions that are valid for a broad class of fading distributions.

In this paper, we analyze the performance of TR signaling for a UWB system with AcR, which can exploit multipath diversity inherent in the environment without the need for stringent

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TABLE I
SIGNALING SCHEMES IN UWB SYSTEMS

Signaling	$d_i^{(\text{PAM},k)}$	$d_i^{(\text{PPM},k)}$	$\{a_j^{(k)}\}$	$\{c_j^{(k)}\}$	T_f
TH-PPM	1	$\{0, 1, \dots, (M-1)\}$	1	$\{0, 1, \dots, N_h - 1\}$	$T_g + N_h T_c + (M-1)\delta \leq T_f$
TH-PAM	$\{\pm 1, \dots, \pm(M-1)\}$	0	1	$\{0, 1, \dots, N_h - 1\}$	$T_g \leq T_f$
DS-PAM	$\{\pm 1, \dots, \pm(M-1)\}$	0	$\{\pm 1\}$	0	$T_f = T_p$
TR	$\{\pm 1\}$	0	$\{\pm 1\}$	$\{0, 1, \dots, N_h - 1\}$	$N_h T_p + T_r \leq 2T_f - T_g$

acquisition and channel estimation. As in [9] and [10], we assume that the multipath channel is resolvable.¹ Our main contribution is the development of an analytical framework, based on the sampling expansion approach, to derive closed-form expression for the BEP of TR signaling with AcR in dense multipath channels [31], [32]. We further extend our analysis to the BEP derivation of modified AcR with noise-averaging. Our methodology does not require the Gaussian approximation and is applicable for any fading scenario, provided that the correlator output signal-to-noise ratio (SNR) can be characterized in terms of a characteristic function (CF). Using our derived BEP expression, we can assess the validity of the conventional Gaussian approximation as a function of the time-bandwidth product and the number of transmitted pulses per symbol. Our results enable the derivation of a computationally simple lower bound on the BEP of TR signaling with AcR. This lower bound is reliable for the BEP range of interest and, thus, allows us to obtain the SNR penalty associated with an AcR, as compared with *all-rake* (ARake) and *partial-rake* (PRake) receivers [33], [34].

This paper continues with Section II, in which the system and channel models are developed. In Section III, the BEP of TR signaling is derived based on the Gaussian approximation and sampling expansion approaches. To illustrate our proposed methodology, we consider Nakagami- m fading channels and present numerical results in Section IV. We present a computationally simple lower bound on the BEP of TR signaling with AcR in Section IV to facilitate performance comparison between the AcR, ARake, and PRake receivers. Finally, Section V comprises concluding remarks.

II. SYSTEM AND CHANNEL MODELS

The transmitted signal of the k th user in a generic UWB system can be described by

$$s^{(k)}(t) = \sum_i d_i^{(\text{PAM},k)} b^{(k)}(t - iN_s T_f - \delta d_i^{(\text{PPM},k)}) \quad (1)$$

where T_f is the average repetition period and $b^{(k)}(t)$ is the k th user's block-modulated signal with symbol interval $N_s T_f$. The data symbol can be modulated using either PAM or PPM. In PAM, $d_i^{(\text{PPM},k)} = 0$ for all i, k , and $d_i^{(\text{PAM},k)}$ is chosen from an alphabet of $\{\pm 1, \pm 3, \dots, \pm(M-1)\}$, where $\log_2 M$ is the number of bits transmitted per symbol. In PPM, $d_i^{(\text{PAM},k)} = 1$ for all i, k , and $d_i^{(\text{PPM},k)}$ is chosen from an alphabet of $\{0, 1, \dots, (M-1)\}$, where δ is the modulation index associated with PPM.

¹Such an assumption may not be always true [29], [30]. However, the resolvable multipath channel may still serve as a reasonable approximation to realistic UWB channels. Therefore, our BEP analysis still provides insights into the performance of TR signaling with AcR.

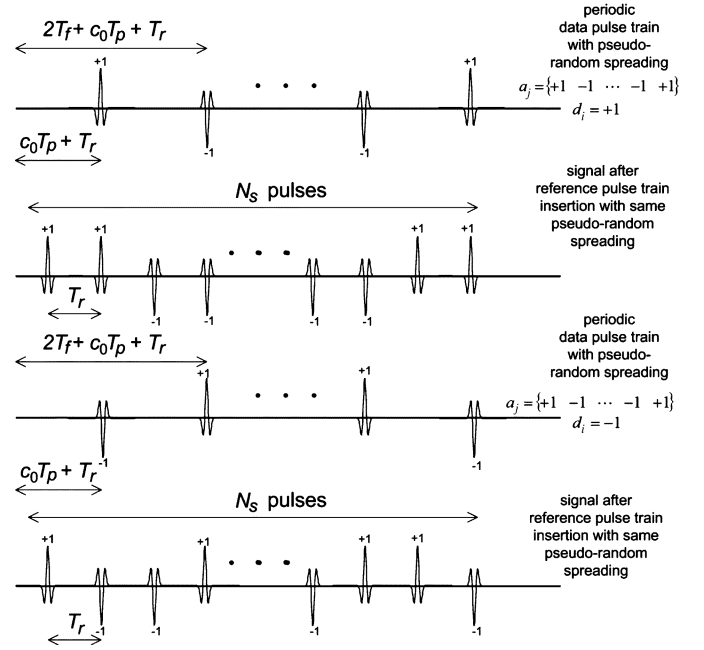


Fig. 1. TR signaling.

Each $b^{(k)}(t)$ -shaped block, containing N_s transmitted signal pulses, can be written as

$$b^{(k)}(t) = \sum_{j=0}^{N_s-1} \sqrt{E_p} a_j^{(k)} p(t - jT_f - c_j^{(k)} T_c) \quad (2)$$

where $p(t)$ is the normalized signal pulse or monocycle with duration T_p and $\int_{-\infty}^{+\infty} |p(t)|^2 dt = 1$. The energy of the transmitted pulse is then $E_p = E_s/N_s$ and the symbol energy is E_s . To enhance the robustness of UWB systems to interference as well as to allow multiple access, DS and/or TH spread spectrum techniques can be used as shown in (2). In DS signaling, $\{a_j^{(k)}\}$ is the bipolar pseudorandom sequence of the k th user. In TH signaling, $\{c_j^{(k)}\}$ is the pseudorandom TH sequence of the k th user, where $c_j^{(k)}$ is an integer in the range $0 \leq c_j^{(k)} < N_h$, N_h is the maximum allowable integer shift, and T_c is an additional TH delay.

Equations (1) and (2) can accommodate several different signaling schemes used in UWB systems, as shown in Table I. As an alternative to LGR systems, we consider TR signaling, as shown in Fig. 1, which can be fully described by (1) and (2). From (1), we first set $d_i^{(\text{PPM},k)} = 0$ and $d_i^{(\text{PAM},k)} = d_i^{(k)} \in \{-1, 1\}$, $\forall i, k$, and decompose the transmitted signal of the user k into a reference signal block $b_r^{(k)}(t)$ and a data modulated signal block $b_d^{(k)}(t)$ as given by

$$s^{(k)}(t) = \sum_i b_r^{(k)}(t - iN_s T_f) + d_i^{(k)} b_d^{(k)}(t - iN_s T_f). \quad (3)$$

Modifying (2), we have

$$b_r^{(k)}(t) = \sum_{j=0}^{\frac{N_s}{2}-1} \sqrt{E_p} a_j^{(k)} p(t - j2T_f - c_j^{(k)} T_p)$$

$$b_d^{(k)}(t) = \sum_{j=0}^{\frac{N_s}{2}-1} \sqrt{E_p} a_j^{(k)} p(t - j2T_f - c_j^{(k)} T_p - T_r) \quad (4)$$

where $T_c = T_p$. In the case of binary signaling, $E_s = E_b$, where E_b is the energy per bit. Note that the inter-pulse delay between each data-modulated monocycle and its corresponding reference monocycle is given by T_r . If the symbol duration is less than the channel coherence time, all these pairs of separated signals will experience the same channel.² We will suppress the index k for notational simplicity in the rest of this paper, since the focus of this paper is on a single link system.

The received signal can be written as

$$r(t) = \int_{-\infty}^{+\infty} h(\tau) s(t - \tau) d\tau + n(t) \quad (5)$$

where $h(t)$ is the impulse response of the channel and $n(t)$ is zero-mean, white Gaussian noise with two-sided power spectral density $N_0/2$. The channel impulse response can be modeled as linear time-invariant $h(t) = \sum_{l=1}^L \alpha_l \delta(t - \tau_l)$, where α_l and τ_l denote, respectively, the attenuation and delay of the l th path, and L is the number of multipath components. We consider the resolvable multipath channel, i.e., $|\tau_l - \tau_j| \geq T_p, \forall l \neq j$, where $\tau_l = \tau_1 + (l-1)T_p$ and α_l are statistically independent random variables (r.v.'s).

In the subsequent section, we will derive the BEP of TR signaling for a broad class of fading channels. However, for numerical illustration, we use Nakagami- m fading. Based on extensive propagation measurement campaigns, it has also been shown that small-scale fading in UWB channels is well modeled by the Nakagami- m distribution [10]. We define the duration of the received UWB pulse as $T_g = T_p + T_d$, where T_d is the maximum excess delay of the channel. To preclude intersymbol interference (ISI) and intrasymbol interference (i.s.i.),³ we assume that $N_h T_p + T_r \leq 2T_f - T_g$ according to Table I and $T_r \geq T_g$.

III. AUTOCORRELATION RECEIVER

A. Receiver Model

With TR signaling, the receiver can exploit the diversity inherent in a multipath channel by using AcR, as shown in Fig. 2. The AcR first passes the received signal through an ideal band-pass zonal filter (BPZF), with bandwidth W and center frequency f_c around the signal band to eliminate the out-of-band noise. If W is wide enough, then the signal spectrum will pass undistorted through the BPZF. Consequently, the ISI and i.s.i.

²Although only two pulses are needed to be within the channel coherence time, the condition that all pulses within the same symbol experience the same channel enables us to extend our analysis to TR signaling with modified AcR, where the channel is assumed to be constant over two symbols. This scheme is addressed in Section III-C.

³ISI and i.s.i. may not always be negligible due to constraints on T_f and data-rate requirements. In this case, our results serve as a lower bound.

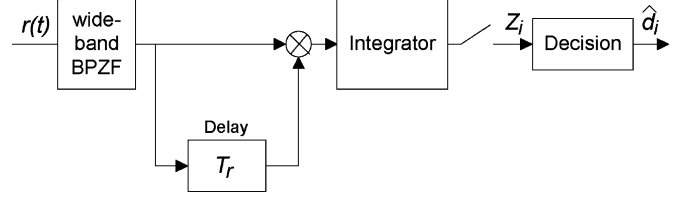


Fig. 2. TR signaling with AcR.

caused by filtering will be negligible. Using (3)–(5), the received signal at the output of the BPZF can be expressed as

$$\tilde{r}(t) = \sum_i \sum_{j=0}^{\frac{N_s}{2}-1} \sum_{l=1}^L \left[\sqrt{E_p} \alpha_l a_j p(t - iN_s T_f - j2T_f - c_j T_p - \tau_l) + \sqrt{E_p} \alpha_l a_j d_i p(t - iN_s T_f - j2T_f - c_j T_p - T_r - \tau_l) \right] + \tilde{n}(t) \quad (6)$$

where $\tilde{r}(t)$ denotes the filtered received signal, and $\tilde{n}(t)$ is a zero-mean, Gaussian random process with autocorrelation function [27]

$$R_{\tilde{n}}(\tau) = WN_0 \text{sinc}(W\tau) \cos(2\pi f_c \tau). \quad (7)$$

Note that when $W \gg 1/T_g$, $R_{\tilde{n}}(t - u)$ in (7) is approximately equal to zero for $|t - u| \geq T_g$. Hence, noise samples separated by more than T_g are assumed to be statistically independent. Note also that the noise samples separated at a multiple of $1/W$ are statistically independent.

The filtered received signal is then passed through a correlator with integration interval T ($T_p \leq T \leq T_g$), as shown in Fig. 2, to collect the received signal energy. The integration interval T determines the amount of multipath and noise energy accumulation, and the actual number of multipath components captured by the AcR is denoted by $L_{CAP} \triangleq \lceil \min\{WT, WT_g\} \rceil$. Without loss of generality, we consider the detection of the data symbol at $i = 0$. Summing all the correlation values corresponding to the $N_s/2$ monocycles, we can obtain the decision statistic required for data detection as

$$Z_{TR} = \sum_{j=0}^{\frac{N_s}{2}-1} \int_{j2T_f+T_r+c_jT_p}^{j2T_f+T_r+c_jT_p+T} \tilde{r}(t) \tilde{r}(t - T_r) dt = Z_1 + Z_2 + Z_3 + Z_4 \quad (8)$$

where each of the terms in (8) is defined and derived in Appendix I. Next, we derive the BEP of TR signaling in a dense multipath channel as a function of the system parameters (W , N_s , and T) using two approaches, namely, the conventional Gaussian approximation and sampling expansion.

B. Conventional Gaussian Approximation Analysis

The conventional approach to analyze the performance of TR signaling is to assume that the distributions of the noise components Z_2 , Z_3 , and Z_4 in (8) are conditionally Gaussian and mutually uncorrelated when conditioned on $\{\alpha_l\}_{l=1}^L$ [25]–[28]. This assumption is valid when the time-bandwidth product or N_s is large. When the time-bandwidth product is large, (8) can

be approximated by a conditional Gaussian r.v. by invoking the central limit theorem [27], [28]. When N_s is large, Z_4 in (48) can also be approximated as Gaussian by the central limit theorem [27]. The mutually uncorrelated assumption is valid when $W \gg 1/T_g$ such that $R_n(t-u)$ in (7) is nonzero for values of t and u when they are approximately equal [25]–[28]. Only when all the above assumptions are valid, can we invoke the Gaussian approximation to derive the conditional BEP for TR signaling with AcR as

$$\mathbb{P}\{e|\gamma_{\text{TR}}\} = Q\left(\sqrt{\frac{\mathbb{E}\left\{Z_{\text{TR}}|\{\alpha_l\}_{l=1}^{L_{\text{CAP}}}, d_0\right\}^2}{\text{var}\left\{Z_{\text{TR}}|\{\alpha_l\}_{l=1}^{L_{\text{CAP}}}\right\}}}\right) \quad (9)$$

where $Q(\cdot)$ is the Gaussian Q -function. Therefore, under the Gaussian approximation, the derivation of the conditional BEP in (9) is reduced to the derivation of the conditional mean and variance. The conditional mean is given by

$$\begin{aligned} \mathbb{E}\left\{Z_{\text{TR}}|\{\alpha_l\}_{l=1}^{L_{\text{CAP}}}, d_0\right\} &= Z_1 \\ &= \frac{E_s d_0}{2} \sum_{l=1}^{L_{\text{CAP}}} \alpha_l^2 \end{aligned} \quad (10)$$

where the second equality is due to Appendix I. The mutual independence assumption of Z_2 , Z_3 , and Z_4 leads to the following result (see Appendix II):

$$\begin{aligned} \text{var}\left\{Z_{\text{TR}}|\{\alpha_l\}_{l=1}^{L_{\text{CAP}}}\right\} &= \text{var}\left\{Z_2|\{\alpha_l\}_{l=1}^{L_{\text{CAP}}}\right\} + \text{var}\left\{Z_3|\{\alpha_l\}_{l=1}^{L_{\text{CAP}}}\right\} \\ &\quad + \text{var}\left\{Z_4|\{\alpha_l\}_{l=1}^{L_{\text{CAP}}}\right\} \\ &\approx \frac{N_0 E_s}{2} \sum_{l=1}^{L_{\text{CAP}}} \alpha_l^2 + \frac{N_s}{4} N_0^2 W T. \end{aligned} \quad (11)$$

Using (9)–(11), we can rewrite the conditional BEP for TR signaling with AcR as

$$\mathbb{P}\{e|\gamma_{\text{TR}}\} = Q\left(\frac{\gamma_{\text{TR}}}{\sqrt{\gamma_{\text{TR}} + \frac{N_s}{4} W T}}\right) \quad (12)$$

where γ_{TR} is the instantaneous received SNR of TR signaling with AcR defined by

$$\gamma_{\text{TR}} \triangleq \frac{E_s}{2N_0} \sum_{l=1}^{L_{\text{CAP}}} \alpha_l^2. \quad (13)$$

From (12) and (13), we can see that the amount of received energy captured by the AcR depends on the time-bandwidth product. Under the resolvable multipath assumption, L_{CAP} increases with WT until $L_{\text{CAP}} = L$. Increasing WT beyond this point will only accumulate more noise energy in the receiver as seen in the denominator of (12).

The BEP of TR signaling can then be obtained by averaging the conditional BEP in (12) as

$$P_{e,\text{TR}} = \int_0^\infty \mathbb{P}\{e|x\} f_{\gamma_{\text{TR}}}(x) dx \quad (14)$$

where $f_{\gamma_{\text{TR}}}(\cdot)$ is the probability density function (pdf) of γ_{TR} . The direct approach to evaluate (14) seems intractable since $\mathbb{P}\{e|x\}$ is written in terms of a definite integral (i.e., Gaussian Q -function) whose limit is an r.v. to be averaged. A common approach to alleviate this problem is to use the alternative expression for the Gaussian Q -function which has previously enabled numerous analyzes of wireless scenarios involving fading channels [35]. Even with this approach, the evaluation of (14) is very difficult, if at all possible, since γ_{TR} appears in both the numerator and denominator of the argument to the Q -function. At this point, one may resort to numerically averaging (14) via a quasi-analytical/experimental approach [26], [27] as suggested originally in [9] or a quasi-analytical/simulation approach [28]. Hence, this motivates us to develop an analytical framework to derive a closed-form expression for the BEP of TR signaling for a broad class of fading channels.

C. Sampling Expansion Analysis

The conditional BEP for TR signaling with AcR can also be found by evaluating

$$\mathbb{P}\{e|\gamma_{\text{TR}}\} = \frac{1}{2} \mathbb{P}\{Z_{\text{TR}} < 0 | d_0 = +1\} + \frac{1}{2} \mathbb{P}\{Z_{\text{TR}} > 0 | d_0 = -1\}. \quad (15)$$

To derive the conditional BEP in (15), we rewrite Z_{TR} in (8) using (3) and (4) as

$$\begin{aligned} Z_{\text{TR}} &= \sum_{j=0}^{\frac{N_s}{2}-1} \int_0^T \left[\check{b}_r(t + j2T_f + c_j T_p) + \tilde{n}(t + j2T_f + c_j T_p) \right] \\ &\quad \times \left[d_0 \check{b}_d(t + j2T_f + c_j T_p + T_r) \right. \\ &\quad \left. + \tilde{n}(t + j2T_f + c_j T_p + T_r) \right] dt \end{aligned} \quad (16)$$

where $\check{b}_r(t) \triangleq (b_r * h * h_{\text{ZPF}})(t)$, $\check{b}_d(t) \triangleq (b_d * h * h_{\text{ZPF}})(t)$, and $h_{\text{ZPF}}(t)$ is the impulse response of the BPZF. Note that if the symbol duration is less than the coherence time, all pairs of separated pulses will experience the same channel, implying that $\check{b}_r(t + j2T_f + c_j T_p) = \check{b}_d(t + j2T_f + c_j T_p + T_r)$ for all $t \in (0, T)$ and c_j . In this case, we can significantly simplify the expression in (16) as follows:

$$\begin{aligned} Z_{\text{TR}} &= \sum_{j=0}^{\frac{N_s}{2}-1} \int_0^T (w_j(t) + \eta_{1,j}(t)) (d_0 w_j(t) + \eta_{2,j}(t)) dt \\ &= \sum_{j=0}^{\frac{N_s}{2}-1} U_j \end{aligned} \quad (17)$$

where we have used $w_j(t) \triangleq \check{b}_r(t + j2T_f + c_j T_p) = \sqrt{E_p} a_j \sum_{l=1}^L \alpha_l p(t - \tau_l)$, $\eta_{1,j}(t) \triangleq \tilde{n}(t + j2T_f + c_j T_p)$ and $\eta_{2,j}(t) \triangleq \tilde{n}(t + j2T_f + c_j T_p + T_r)$ defined over the interval $[0, T]$. Note that since the noise samples are taken at an interval larger than T_g apart, they are essentially independent, regardless of c_j . Hence, no further assumption on c_j is required in our analysis. We further observe that U_j is simply the integrator output of the j th received modulated monocycle.

To enable the BEP analysis of TR signaling, we now transform a generic signal $u(t)$ into a finite dimensional random

vector using the classical signal space approach.⁴ In this approach, the received signals are considered as elements in an infinite dimensional Hilbert space [38]. Since the received signals are band-limited and effectively time-limited, we can use the sampling functions as the set of orthonormal functions to project the received waveform onto the subspace of band-limited functions with approximate dimensionality $2WT$ [39].

We begin with the introduction of the Sampling Theorem [40]–[42]. Recall that a real signal $u(t)$ with a lowpass bandwidth of $W/2$ can be written as

$$u(t) = \int_{-\frac{W}{2}}^{\frac{W}{2}} \hat{u}(f) e^{j2\pi ft} df \quad (18)$$

where $\hat{u}(f) \in L_2[-(W/2), W/2]$ and $L_2[-(W/2), W/2]$ denotes the Hilbert space of all square-integrable real-valued functions with support $[-(W/2), W/2]$ and $W > 0$. It is well known that $u(t)$ can be represented by the Whittaker–Shannon–Kotel’nikov sampling series [40]–[42] as

$$u(t) = \sum_{m=-\infty}^{\infty} u\left(\frac{m}{W}\right) \text{sinc}(Wt - m). \quad (19)$$

The series in (19) is also known as a cardinal series since the sampling functions are the cardinal sine functions or sinc functions.⁵ Instead of using (19), we can approximate $u(t)$ by truncating the cardinal series to about WT terms with a truncation error that scales in $O(1/\sqrt{WT})$ [43]. Thus, for large WT , the space of band-limited and effectively time-limited signals is asymptotically WT -dimensional [38], [44].⁶

Applying this “dimensionality theorem” [38], [44], we consider the computation of the following integral:

$$z(\tau) = \int_0^T u(t)v(t + \tau)dt \quad (20)$$

where $u(t)$ and $v(t)$ are real bandwidth-limited functions. Employing the truncated cardinal series, $u(t)$ and $v(t)$ can be approximated by WT -tuple real vectors in the subspace of band-limited functions. Since this signal space has an assigned inner product, we can then rewrite (20) as

$$z_n \approx \frac{1}{W} \sum_{m=1}^{WT} u_m v_{m+n} \quad (21)$$

where u_m and v_m are samples of $u(t)$ and $v(t)$, respectively, sampled at the Nyquist rate with a sampling interval of $\Delta = 1/W$ [21], [39], [45]. The total number of real samples over $[0, T]$ is then $T/\Delta = WT$. Alternatively, we can use prolate

⁴Our approach follows the general philosophy of solving difficult problems in the transform domain [36], [37].

⁵Note that the set of orthonormal functions in the expansion of (19) is $\{\sqrt{W} \text{sinc}(Wt - m)\}$ with corresponding coefficients $(1/\sqrt{W})u(m/W)$.

⁶Since UWB channel can contain several hundreds paths of significant strength [9], [10], the number of degrees of freedom available in a given time-bandwidth product is also large and, thus, allow us to apply the “dimensionality theorem” [38], [44] to represent the received bandpass signals as $2WT$ -tuple real vectors.

spheroidal wave functions as the set of orthonormal functions [46]–[48]. In fact, such expansion was used to study the performance of TR signaling in [15].

Since the received signal is a real bandpass signal of bandwidth W , one can think of it in terms of a baseband equivalent complex model. In this case, the signal is complex and bandlimited to $W/2$. The sampling theorem then states that this must be sampled at a sampling frequency greater than or equal to W . This gives WT complex dimensions or $2WT$ real dimensions [45]. Hence, we can apply the truncated cardinal series and the integral approximation (21) with $n = 0$ to represent U_j by $2WT$ samples as follows:

$$U_j = \frac{1}{W} \sum_{m=1}^{2WT} (d_0 w_{j,m}^2 + w_{j,m} \eta_{2,j,m} + d_0 w_{j,m} \eta_{1,j,m} + \eta_{1,j,m} \eta_{2,j,m}) \quad (22)$$

where the m th sample of $w_j(t)$, $\eta_{1,j}(t)$ and $\eta_{2,j}(t)$ in (17) are, respectively, $w_{j,m}$, $\eta_{1,j,m}$, and $\eta_{2,j,m}$ in the interval $[0, T]$. Conditioned on d_0 , we can express (22) in the form of a summation of squares

$$U_{j|d_0=+1} = \sum_{m=1}^{2WT} \left[\left(\frac{1}{\sqrt{W}} w_{j,m} + \beta_{1,j,m} \right)^2 - \beta_{2,j,m}^2 \right] \quad (23)$$

$$U_{j|d_0=-1} = \sum_{m=1}^{2WT} \left[- \left(\frac{1}{\sqrt{W}} w_{j,m} - \beta_{2,j,m} \right)^2 + \beta_{1,j,m}^2 \right] \quad (24)$$

where $\beta_{1,j,m}$ and $\beta_{2,j,m}$ are statistically independent Gaussian r.v.’s given by

$$\begin{aligned} \beta_{1,j,m} &= \frac{1}{2\sqrt{W}} (\eta_{2,j,m} + \eta_{1,j,m}) \\ \beta_{2,j,m} &= \frac{1}{2\sqrt{W}} (\eta_{2,j,m} - \eta_{1,j,m}) \end{aligned} \quad (25)$$

with variance $\sigma_{\text{TR}}^2 = N_0/4$. Recall from the definition of $w_j(t)$ that the contribution of the spreading code a_j is embedded in $w_{j,m}$. From (23), we can observe that when conditioned on the channel, $U_{j|d_0=+1}$ has the same pdf for $a_j = +1$ and $a_j = -1$. Due to this symmetry and the fact that a_j is equally probable, we can assume without loss of generality that $a_j = +1$ in the following analysis. For notational simplicity, we now define the normalized r.v.’s Y_1 , Y_2 , Y_3 , and Y_4 as

$$\begin{aligned} Y_1 &\triangleq \frac{1}{2\sigma_{\text{TR}}^2} \sum_{j=0}^{\frac{N_s}{2}-1} \sum_{m=1}^{2WT} \left(\frac{1}{\sqrt{W}} w_{j,m} + \beta_{1,j,m} \right)^2 \\ Y_2 &\triangleq \frac{1}{2\sigma_{\text{TR}}^2} \sum_{j=0}^{\frac{N_s}{2}-1} \sum_{m=1}^{2WT} \beta_{2,j,m}^2 \\ Y_3 &\triangleq \frac{1}{2\sigma_{\text{TR}}^2} \sum_{j=0}^{\frac{N_s}{2}-1} \sum_{m=1}^{2WT} \left(\frac{1}{\sqrt{W}} w_{j,m} - \beta_{2,j,m} \right)^2 \\ Y_4 &\triangleq \frac{1}{2\sigma_{\text{TR}}^2} \sum_{j=0}^{\frac{N_s}{2}-1} \sum_{m=1}^{2WT} \beta_{1,j,m}^2. \end{aligned} \quad (26)$$

Conditioned on the channel, Y_1 and Y_3 are noncentral chi-squared r.v.’s with $N_s WT$ degrees of freedom, whereas Y_2 and Y_4 are central chi-squared r.v.’s with the same degrees of

freedom as Y_1 and Y_3 . Both Y_1 and Y_3 have the same noncentrality parameter given by

$$\mu_{\text{TR}} \triangleq \frac{1}{2\sigma_{\text{TR}}^2} \sum_{j=0}^{\frac{N_s}{2}-1} \sum_{m=1}^{2WT} \frac{w_{j,m}^2}{W}. \quad (27)$$

In a dense multipath channel,⁷ L is approximately equal to WT_g [7]–[9], [33] and (27) becomes

$$\mu_{\text{TR}} = \frac{1}{2\sigma_{\text{TR}}^2} \sum_{j=0}^{\frac{N_s}{2}-1} \int_0^T w_j^2(t) dt = \frac{E_s}{N_0} \sum_{l=1}^{L_{\text{CAP}}} \alpha_l^2. \quad (28)$$

Note that $\gamma_{\text{TR}} = \mu_{\text{TR}}/2$. Due to the symmetry, we simply need to calculate the BEP conditioned on $d_0 = +1$ to obtain the BEP. Therefore, the pdfs of Y_1 and Y_2 conditioned on γ_{TR} are given by

$$f_{Y_1|\gamma_{\text{TR}}}(y_1) = f_{\text{NC}}(y_1, \mu_{\text{TR}}, q_{\text{TR}}) \quad (29)$$

$$f_{Y_2|\gamma_{\text{TR}}}(y_2) = f_{\text{C}}(y_2, q_{\text{TR}}) \quad (30)$$

where $q_{\text{TR}} = N_s WT/2$. We have defined the following pdfs for notational convenience:

$$f_{\text{NC}}(y, \mu, n) \triangleq e^{-(y+\mu)} \left(\frac{y}{\mu}\right)^{\frac{(n-1)}{2}} I_{n-1}(2\sqrt{y\mu}), \quad y \geq 0 \quad (31)$$

$$f_{\text{C}}(y, n) \triangleq \frac{y^{(n-1)}}{(n-1)!} \exp(-y), \quad y \geq 0 \quad (32)$$

where $I_{n-1}(\cdot)$ is the $(n-1)$ th order Bessel function of the first kind, and $f_{\text{NC}}(y, \mu, n)$ and $f_{\text{C}}(y, n)$ are, respectively, the pdfs of the noncentral and central chi-squared r.v.'s with $2n$ degrees of freedom. Using (29) and (30), the conditional BEP in (15) becomes

$$\begin{aligned} & \mathbb{P}\{e|\gamma_{\text{TR}}\} \\ &= \mathbb{P}\{Y_1 < Y_2 | d_0 = +1\} \\ &= \frac{e^{-\gamma_{\text{TR}}}}{2^{q_{\text{TR}}}} \sum_{i=0}^{q_{\text{TR}}-1} \frac{(\gamma_{\text{TR}})^i}{i!} \sum_{k=i}^{q_{\text{TR}}-1} \frac{1}{2^k} \frac{(k+q_{\text{TR}}-1)!}{(k-i)!(q_{\text{TR}}+i-1)!} \end{aligned} \quad (33)$$

where the detailed derivation of (33) can be found in Appendix III.

Recall that the conditional BEP in (12), obtained by the Gaussian approximation, has the output SNR appearing in both

⁷Note that in short-range indoor environments, L can be large but significantly less than WT_g due to insufficient scattering. In this case, our results will give a lower bound.

the numerator and denominator of the Gaussian Q -function, which makes the BEP analysis in fading channels intractable. In contrast, the form in (33) is desirable as it enables the averaging of $\mathbb{P}\{e|\gamma_{\text{TR}}\}$ with respect to γ_{TR} as follows:

$$\begin{aligned} P_{e,\text{TR}} &= \mathbb{E}\{\mathbb{P}\{e|\gamma_{\text{TR}}\}\} \\ &= \frac{1}{2^{q_{\text{TR}}}} \sum_{i=0}^{q_{\text{TR}}-1} \left[\frac{\mathbb{E}\{(\gamma_{\text{TR}})^i e^{-\gamma_{\text{TR}}}\}}{i!} \right] \\ &\quad \times \sum_{k=i}^{q_{\text{TR}}-1} \frac{1}{2^k} \frac{(k+q_{\text{TR}}-1)!}{(k-i)!(q_{\text{TR}}+i-1)!} \\ &= \frac{1}{2^{q_{\text{TR}}}} \left[\sum_{i=0}^{q_{\text{TR}}-1} \frac{(-j)^i}{i!} \frac{d^i}{dv^i} \psi_{\gamma_{\text{TR}}}(jv) \right]_{jv=-1} \\ &\quad \times \sum_{k=i}^{q_{\text{TR}}-1} \frac{1}{2^k} \frac{(k+q_{\text{TR}}-1)!}{(k-i)!(q_{\text{TR}}+i-1)!} \\ &\triangleq P_e(\psi_{\gamma_{\text{TR}}}(jv), q_{\text{TR}}) \end{aligned} \quad (34)$$

where $\psi_{\gamma_{\text{TR}}}(jv) \triangleq \mathbb{E}\{e^{jv\gamma_{\text{TR}}}\}$ is the CF of γ_{TR} and $P_e(\psi_{\gamma_{\text{TR}}}(jv), q_{\text{TR}})$ is defined for notational convenience. Under the resolvable multipath assumption, multipath components are statistically independent and $\psi_{\gamma_{\text{TR}}}(jv)$ is given by

$$\psi_{\gamma_{\text{TR}}}(jv) = \prod_{l=1}^{L_{\text{CAP}}} \psi_l \left(jv \frac{E_s}{2N_0} \right) \quad (35)$$

where $\psi_l(jv)$ is the CF of α_l^2 . For a wide class of channel statistics such as Nakagami, Rice, and Rayleigh [35], $\psi_l(jv)$ has a well-known closed-form expression. Thus, (34) gives a closed-form expression for the BEP of TR signaling with AcR. For numerical illustration, we consider independent Nakagami- m fading channels with uniform power dispersion profile (PDP), which serves as a benchmark. In this case, the BEP of TR signaling with AcR becomes

$$P_{e,\text{TR}} = P_e \left(\left(1 - jv \frac{E_s}{2LmN_0} \right)^{-mL_{\text{CAP}}}, q_{\text{TR}} \right). \quad (36)$$

Next, we extend the above analysis to derive the BEP of TR signaling with modified AcR. The AcR performance can be improved by averaging over the $N_s/2$ received reference pulses from the previous symbol, to be used as an estimate of the channel [27]. This, however, assumes that the channel remains constant over two symbols. The decision statistics of this modified AcR for TR signaling is given by (37) shown at the bottom of the page, where the terms inside the parentheses can be thought of as an improved channel estimate. The variance of $\eta_{1,j,m}/\sqrt{W}$ in (22) is now reduced to N_0/N_s due to the noise

$$Z_{\text{ATR}} = \sum_{j=0}^{\frac{N_s}{2}-1} a_j \int_{j2T_f+T_r+c_jT_p}^{j2T_f+T_r+c_jT_p+T} \tilde{r}(t) \left[\frac{2}{N_s} \sum_{k=-j}^{\frac{N_s}{2}-1-j} a_{j+k} \tilde{r}(t - (N_s - 2k)T_f - (c_j - c_{j+k})T_p - T_r) \right] dt \quad (37)$$

averaging effect in (37). As a result, the variance σ_{ATR}^2 of $\beta_{1,j,m}$ and $\beta_{2,j,m}$ becomes

$$\sigma_{\text{ATR}}^2 = \frac{\text{var}\{\eta_{2,j,m}\} + \text{var}\{\eta_{1,j,m}\}}{4W} = \frac{N_0(N_s + 2)}{8N_s} \quad (38)$$

and the noncentrality parameter of Y_1 in (26) becomes

$$\begin{aligned} \mu_{\text{ATR}} &\triangleq \frac{1}{2\sigma_{\text{ATR}}^2} \sum_{j=0}^{\frac{N_s}{2}-1} \sum_{m=1}^{2WT} \frac{w_{j,k}^2}{W} \\ &= \frac{2}{1 + \frac{2}{N_s}} \left(\frac{E_s}{N_0} \sum_{l=1}^{L_{\text{CAP}}} \alpha_l^2 \right). \end{aligned} \quad (39)$$

The instantaneous received SNR of TR signaling with modified AcR is then given by

$$\gamma_{\text{ATR}} = \frac{\mu_{\text{ATR}}}{2} = \frac{2}{1 + \frac{2}{N_s}} \gamma_{\text{TR}} \quad (40)$$

and the derivation of the BEP of TR signaling with modified AcR follows directly from (33) and (34), except that $\psi_{\gamma_{\text{TR}}}(jv)$ is now replaced by $\psi_{\gamma_{\text{ATR}}}(jv)$ having an expansion analogous to (35). As a result, the BEP of TR signaling with modified AcR becomes

$$P_{e,\text{ATR}} = P_e(\psi_{\gamma_{\text{ATR}}}(jv), q_{\text{TR}}). \quad (41)$$

For independent Nakagami channels with uniform PDP, the BEP of TR signaling with modified AcR is given by

$$P_{e,\text{ATR}} = P_e \left(\left(1 - jv \frac{E_s}{\left(1 + \frac{2}{N_s}\right) LmN_0} \right)^{-mL_{\text{CAP}}}, q_{\text{TR}} \right). \quad (42)$$

Comparing (36) to (42), we can observe that the modified AcR with large N_s ($N_s \gg 2$) can at most improve the average SNR per path by a factor of 2.

IV. NUMERICAL RESULTS AND DISCUSSIONS

A. BEP Performance

In this section, some numerical results are given to illustrate the BEP performance of TR signaling with AcR using the Gaussian approximation and the sampling expansion approach. For simplicity, we have restricted results to independent Nakagami- m fading channels with uniform PDP and $L = 40$. Figs. 3 and 4 show the BEP performance of TR signaling with AcR when $N_s = 2$ and $N_s = 16$, respectively. Both figures show that the performance of TR signaling with AcR generally improves as WT increases. This is expected since more multipath components are captured by the receiver, resulting in an increase in diversity order, as well as energy capture. It can also be seen that results based on the Gaussian approximation can differ from our closed-form BEP expression in (36). Furthermore, the accuracy of the Gaussian approximation improves as WT and N_s increase as explained in Section III-B.

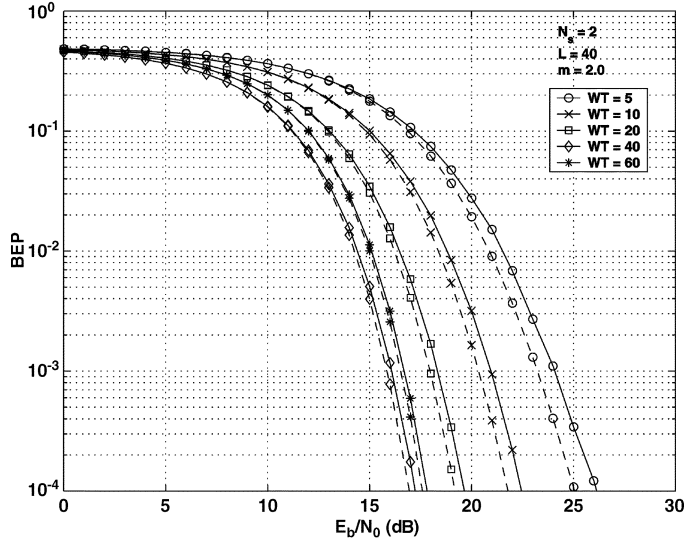


Fig. 3. BEP performance of TR signaling with AcR in independent Nakagami- m fading channels with uniform PDP, $m = 2.0$, and $N_s = 2$. The solid and dashed lines denote the Gaussian approximation and sampling expansion, respectively.

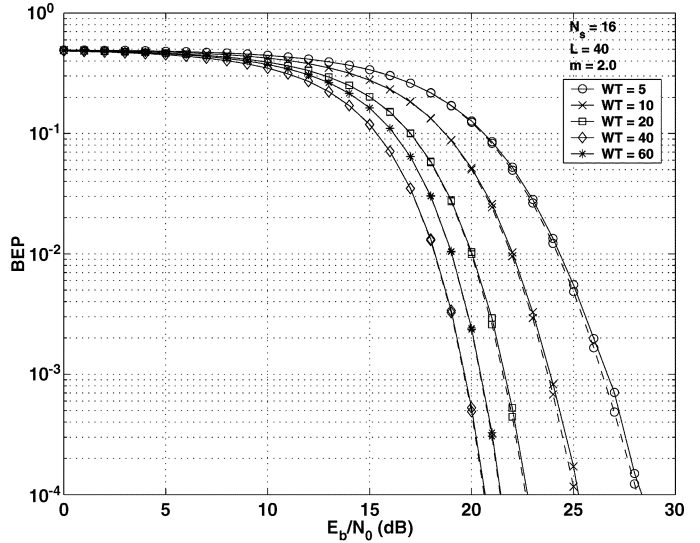


Fig. 4. BEP performance of TR signaling with AcR in independent Nakagami- m fading channels with uniform PDP, $m = 2.0$, and $N_s = 16$. The solid and dashed lines denote the Gaussian approximation and sampling expansion, respectively.

The effect of WT on the BEP performance of TR signaling is shown in Fig. 5. It can be observed that the BEP decreases with WT until it reaches the optimum value of 40, which is equal to L .⁸ This behavior, which is more pronounced at high E_b/N_0 , can be explained by the fact that the loss due to noise accumulation is less than the gain of capturing more multipath energy as WT increases. However, increasing WT beyond the optimum value will only accumulate more noise energy, as reflected in the increase of BEP after $WT = 40$. In addition, we can also see that the diversity gain is larger for the higher fading severity index m , especially when E_b/N_0 is high.

⁸Note that the optimum integration interval of AcR is not always equal to L , and depends on the PDP of the channel.

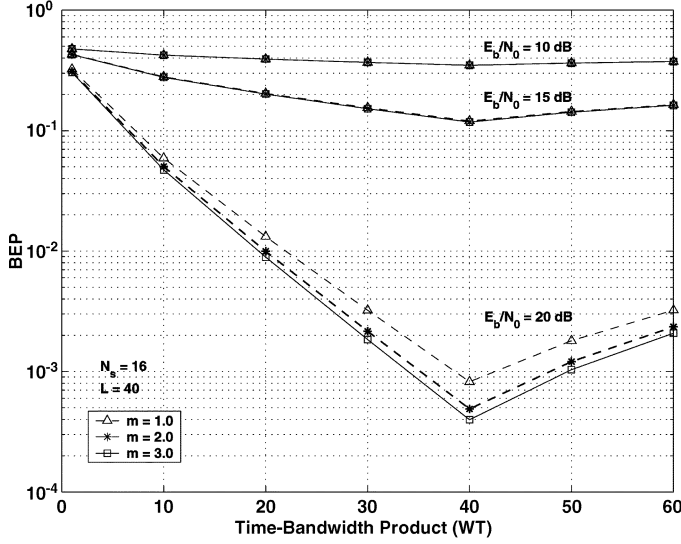


Fig. 5. Effect of integration interval T of AcR on the performance of TR signaling in independent Nakagami- m fading channels with uniform PDP.

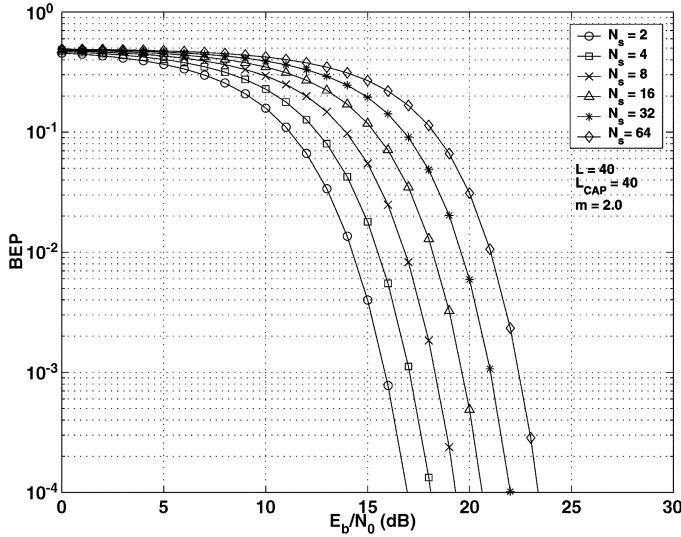


Fig. 6. Effect of N_s on the performance of TR signaling with AcR in independent Nakagami- m fading channels with uniform PDP, $m = 2.0$, and $L_{CAP} = L$.

In Fig. 6, the effect of N_s on the BEP performance of TR signaling with AcR is considered using the closed-form expression in (36) with $L_{CAP} = L$. For a fixed E_b/N_0 , increasing N_s is equivalent to increasing the degrees of freedom of q_{TR} in (34), which leads to more noise energy accumulation. This can be seen as the gradual performance degradation as N_s increases in Fig. 6. In order to improve the performance of TR signaling with AcR, the modified AcR can be used and the BEP performance comparison is shown in Fig. 7 using (36) and (42). From Fig. 7, it can be seen that the modified AcR performs better than the AcR by about 1 and 3 dB, respectively, for $N_s = 4$ and $N_s = 32$. This verified our claim in (42) that the achievable performance gain of modified AcR over AcR is at most 3 dB for large N_s .

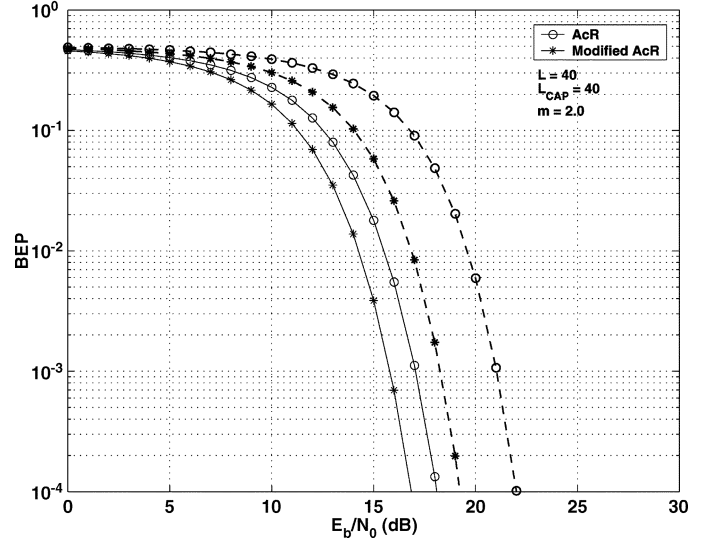


Fig. 7. BEP performance of TR signaling with AcR and modified AcR in independent Nakagami- m fading channels with uniform PDP, $m = 2.0$, and $L_{CAP} = L$. The solid and dashed lines indicate $N_s = 4$, and $N_s = 32$, respectively.

B. SNR Penalty

With the closed-form expression in (34), one can now answer the following question: What is the SNR penalty associated with TR signaling with AcR when compared to ARake and PRake receivers? In the following, we provide some numerical results and a computationally simple lower bound on (34) to quantify the SNR penalty associated with TR signaling with AcR.

The ideal Rake receiver with full diversity is known as the ARake receiver [33], whereas the PRake receiver refers to a lower complexity Rake receiver that combines only the first incoming L_p multipath components [34]. Note that both receivers assume perfect channel estimation. By using the alternative expression for Gaussian Q -function, the BEP of binary phase-shift keying (BPSK) for a PRake receiver in independent Nakagami channels with uniform PDP, is given by [35]

$$P_{e,PRake} = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \left(1 + \frac{\bar{\gamma}}{m \sin^2 \theta} \right)^{-mL_p} d\theta \quad (43)$$

where $\bar{\gamma} = E_s/LN_0$. When $L_p = L$, (43) simply becomes the BEP of BPSK for the ARake receiver.

From (33), we can derive a simple lower bound on the BEP of TR signaling with AcR by using a first order Taylor series approximation of (33) about $\mathbb{E}\{\gamma_{TR}\}$ to obtain

$$P_{e,TR} \geq \frac{\exp(-\mathbb{E}\{\gamma_{TR}\})}{2^{q_{TR}}} \sum_{i=0}^{q_{TR}-1} \frac{(\mathbb{E}\{\gamma_{TR}\})^i}{i!} \times \sum_{k=i}^{q_{TR}-1} \frac{1}{2^k} \frac{(k+q_{TR}-1)!}{(k-i)!(q_{TR}+i-1)!} \quad (44)$$

where $\mathbb{E}\{\gamma_{TR}\} = E_s/2N_0$. In Fig. 8, the above lower bound is plotted against the exact BEP in (36) when $m = 2.0$ and

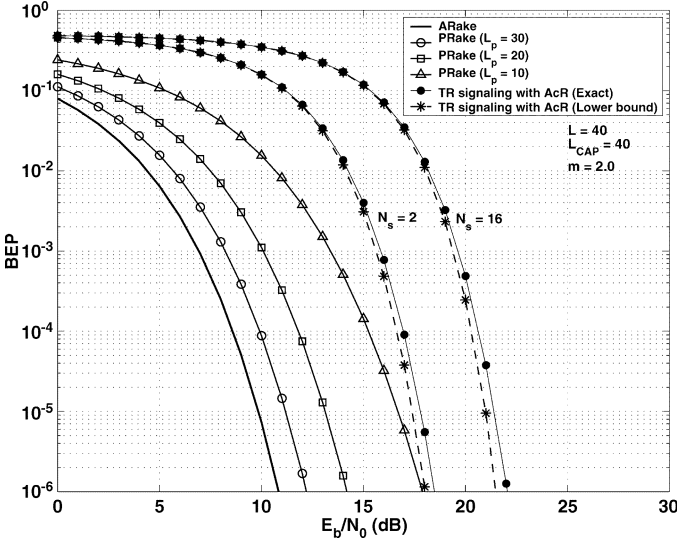


Fig. 8. BEP performance comparison between TR signaling with AcR, PRake and ARake receivers in independent Nakagami- m fading channels with uniform PDP, $m = 2.0$, and $L_{CAP} = L$. The dashed lines indicate the lower bound in (44).

$L_{CAP} = L$.⁹ In addition, the BEP performance curves of the ARake and PRake receivers using (43) are included to obtain the SNR penalty associated with AcR. It can be observed that the lower bound in (44) is quite close to the exact expression in (36) for the BEP range of interest. Hence, this computationally simple lower bound can be used for assessing the SNR penalty associated with an AcR, as compared with ARake and PRake receivers. For example, it can be seen in Fig. 8 that the AcR suffers a SNR penalty of about 8.8 and 12.6 dB, respectively, at $\text{BEP} = 10^{-3}$ for $N_s = 2$ and $N_s = 16$ with respect to the ARake receiver. However, this SNR penalty decreases with respect to PRake receiver when L_p decreases. At $\text{BEP} = 10^{-3}$, the SNR penalty associated with AcR, as compared with PRake receiver with $L_p = 10$ is about 2.1 and 5.9 dB, respectively, for $N_s = 2$ and $N_s = 16$.

V. CONCLUSION

In this paper, we analyzed the performance of TR signaling in conjunction with a simple AcR in dense multipath UWB channels. We derived the BEP expression for TR signaling with AcR via the Gaussian approximation and sampling expansion approaches. We showed the limitation of the Gaussian approximation whose validity depends on the assumption of a large time-bandwidth product or large number of transmitted pulses per symbol. Based on the sampling expansion approach, we developed an analytical framework to derive a closed-form expression for the BEP of TR signaling with AcR for a broad class of fading channels. We extended our methodology to derive the BEP of TR signaling with modified AcR. Furthermore, we compared the performance of TR signaling with AcR with that of the ARake and PRake receivers using our computationally simple

⁹Fig. 8 shows the BEP as low as 10^{-6} only to illustrate the behavior of the lower bound; these extremely low BEPs are not practical, especially for wireless mobile communications.

lower bound. This allowed us to obtain the SNR penalty associated with TR signaling with an AcR, as compared to ARake and PRake receivers for the BEP range of interest. We have deliberately restricted our attention to binary signals to accentuate the development of our analytical framework based on sampling expansion. Results can be extended to the M -ary case.

APPENDIX I OUTPUT STATISTICS OF THE AcR

In this appendix, we expand the output statistics of Z_{TR} , as

$$\begin{aligned}
 Z_1 &\triangleq \sum_{j=0}^{N_s/2-1} \int_{-\infty}^{\infty} \int_{j2T_f+T_r+c_jT_p}^{j2T_f+T_r+c_jT_p+T} [h(\tau)s(t-\tau)h(\tau) \\
 &\quad \times h(\tau)s(t-T_r-\tau)] dt d\tau \\
 &= \sum_{j=0}^{N_s/2-1} \left[\sum_{l=1}^{L_{CAP}} \alpha_l^2 \int_{j2T_f+c_jT_p}^{j2T_f+c_jT_p+T} s(t-\tau_l)s(t+T_r-\tau_l) dt \right. \\
 &\quad \left. + \sum_{l=1}^{L_{CAP}} \sum_{m=1}^{L_{CAP}} \alpha_l \alpha_m \int_{j2T_f+c_jT_p}^{j2T_f+c_jT_p+T} s(t-\tau_l)s(t+T_r-\tau_m) dt \right] \\
 &= \frac{E_s}{2} d_0 \sum_{l=1}^{L_{CAP}} \alpha_l^2 \quad (45) \\
 Z_2 &\triangleq \sum_{j=0}^{N_s/2-1} \int_{-\infty}^{\infty} \int_{j2T_f+T_r+c_jT_p}^{j2T_f+T_r+c_jT_p+T} h(\tau)s(t-\tau)\tilde{n}(t-T_r) dt d\tau \\
 &= \sum_{j=0}^{N_s/2-1} \sum_{l=1}^{L_{CAP}} \alpha_l \int_{j2T_f+c_jT_p}^{j2T_f+c_jT_p+T} s(t+T_r-\tau_l)\tilde{n}(t) dt \quad (46) \\
 Z_3 &\triangleq \sum_{j=0}^{N_s/2-1} \int_{-\infty}^{\infty} \int_{j2T_f+T_r+c_jT_p}^{j2T_f+T_r+c_jT_p+T} h(\tau)s(t-T_r-\tau)\tilde{n}(t) dt d\tau \\
 &= \sum_{j=0}^{N_s/2-1} \sum_{l=1}^{L_{CAP}} \alpha_l \int_{j2T_f+c_jT_p}^{j2T_f+c_jT_p+T} s(t-\tau_l)\tilde{n}(t+T_r) dt \quad (47) \\
 Z_4 &\triangleq \sum_{j=0}^{N_s/2-1} \int_{j2T_f+T_r+c_jT_p}^{j2T_f+T_r+c_jT_p+T} \tilde{n}(t)\tilde{n}(t-T_r) dt \\
 &= \sum_{j=0}^{N_s/2-1} \int_{j2T_f+c_jT_p}^{j2T_f+c_jT_p+T} \tilde{n}(t)\tilde{n}(t+T_r) dt. \quad (48)
 \end{aligned}$$

APPENDIX II

CONDITIONAL VARIANCES OF THE OUTPUT OF AcR

In this appendix, we derive the conditional variances of Z_2 and Z_3 , as shown in (49) and (50) at the top of the next page.

Since the noise components are zero-mean, jointly Gaussian r.v.'s, and by using the result for the fourth-moment of jointly Gaussian random variables [38], the conditional variance of Z_4

$$\begin{aligned} \text{var} \left\{ Z_2 | \{\alpha_l\}_{l=1}^{L_{\text{CAP}}} \right\} &= \sum_{j=0}^{\frac{N_s}{2}-1} \sum_{j'=0}^{\frac{N_s}{2}-1} \sum_{l=1}^{L_{\text{CAP}}} \alpha_l^2 \int_{j2T_f+c_jT_p}^{j2T_f+c_jT_p+T} \int_{j'2T_f+c_{j'}T_p}^{j'2T_f+c_{j'}T_p+T} s(t+T_f-\tau_l)s(u+T_f-\tau_l)R_n^\sim(t-u)dtdu \\ &\simeq \frac{E_s}{4} N_0 \sum_{l=1}^{L_{\text{CAP}}} \alpha_l^2 \end{aligned} \quad (49)$$

$$\begin{aligned} \text{var} \left\{ Z_3 | \{\alpha_l\}_{l=1}^{L_{\text{CAP}}} \right\} &= \sum_{j=0}^{\frac{N_s}{2}-1} \sum_{j'=0}^{\frac{N_s}{2}-1} \sum_{l=1}^{L_{\text{CAP}}} \alpha_l^2 \int_{j2T_f+c_jT_p}^{j2T_f+c_jT_p+T} \int_{j'2T_f+c_{j'}T_p}^{j'2T_f+c_{j'}T_p+T} s(t-\tau_l)s(u-\tau_l)R_n^\sim(t-u)dtdu \\ &\simeq \frac{E_s}{4} N_0 \sum_{l=1}^{L_{\text{CAP}}} \alpha_l^2 \end{aligned} \quad (50)$$

$$\begin{aligned} \text{var} \left\{ Z_4 | \{\alpha_l\}_{l=1}^{L_{\text{CAP}}} \right\} &= \sum_{j=0}^{\frac{N_s}{2}-1} \sum_{j'=0}^{\frac{N_s}{2}-1} \int_{j2T_f+c_jT_p}^{j2T_f+c_jT_p+T} \int_{j'2T_f+c_{j'}T_p}^{j'2T_f+c_{j'}T_p+T} R_n^2(t-u) + R_n^\sim(t-u-T_f)R_n^\sim(t-u+T_f)dtdu \\ &\simeq \frac{N_s}{4} N_0^2 WT \end{aligned} \quad (51)$$

$$\begin{aligned} \mathbb{P}\{Y_1 < Y_2\} &= \int_0^\infty 2r_1^n e^{-(r_1^2+\mu)} \left(\frac{1}{\mu}\right)^{\frac{(n-1)}{2}} I_{n-1}(2r_1\sqrt{\mu}) e^{-r_1^2} \sum_{i=0}^{n-1} \frac{r_1^{2i}}{i!} dr_1 \\ &= e^{-\mu} \sum_{i=0}^{n-1} \frac{2}{i!} \left(\frac{1}{\mu}\right)^{\frac{(n-1)}{2}} \int_0^\infty r_1^{n+2i} e^{-2r_1^2} I_{n-1}(2r_1\sqrt{\mu}) dr_1 \end{aligned} \quad (55)$$

$$\int_0^\infty x^{n+2i} e^{-2x^2} I_{n-1}(2x\sqrt{\mu}) dx = \frac{1}{2^n} \left[\frac{i!(4\mu)^{\frac{(n-1)}{2}}}{2^{n+i}} \right] e^{\frac{\mu}{2}} \sum_{k=0}^i \frac{(i+n-1)!}{(i-k)!(n-1+k)!} \frac{(\frac{\mu}{2})^k}{k!} \quad (56)$$

can be derived as shown in (51) at the top of the page. Note that in the above derivations, we have assumed that $W \gg 1/T_g$, so that the autocorrelation function $R_n^\sim(t-u)$ is nonzero for values of t and u which are approximately equal.

APPENDIX III DERIVATION OF $\mathbb{P}\{Y_1 < Y_2\}$

In this appendix, we derive the expression for $\mathbb{P}\{Y_1 < Y_2\}$, where Y_1 and Y_2 are defined in (26) with pdfs $f_{\text{NC}}(y_1, \mu, n)$ and $f_{\text{C}}(y_2, n)$, respectively. Consider two new r.v.'s R_1 and R_2 , where $R_1 = \sqrt{Y_1}$ and $R_2 = \sqrt{Y_2}$, the pdfs of R_1 and R_2 can be found by using transformation of r.v. as follows:

$$f_{R_1}(r_1) = 2r_1 f_{\text{NC}}(r_1^2, \mu, n) \quad (52)$$

$$f_{R_2}(r_2) = 2r_2 f_{\text{C}}(r_2^2, n). \quad (53)$$

Using (53), $F_{R_2}(r_2)$ can be obtained as

$$F_{R_2}(r_2) = 1 - e^{-r_2^2} \sum_{i=0}^{n-1} \frac{r_2^{2i}}{i!}. \quad (54)$$

By using (52) and (54), we can rewrite $\mathbb{P}\{Y_1 < Y_2\}$, as shown in (55) at the top of this page. The following integral given in (56) at the top of this page can be found in [49]. By substituting (56) into (55), we have

$$\mathbb{P}\{Y_1 < Y_2\} = \frac{e^{-\frac{\mu}{2}}}{2^n} \sum_{i=0}^{n-1} \frac{(\frac{\mu}{2})^i}{i!} \sum_{k=i}^{n-1} \frac{1}{2^k} \frac{(k+n-1)!}{(k-i)!(n+i-1)!}. \quad (57)$$

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