ITERATIVE CARRIER SYNCHRONIZATION TECHNIQUES IN TRANSMISSION SYSTEMS PROTECTED BY A POWERFUL ERROR-CORRECTING CODE

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Abstract - **This contribution briefly summarizes the authors' previous work on iterative carrier synchronization in transmission systems protected by a powerful error-correcting code. It is shown how maximum-likelihood estimation theory can be used to derive code-aided synchronization structures that are well suited for application in receivers with iterative maximum-a-posteriori bit detection. Simulation results for a turbo-coded system indicate that the proposed joint carrier synchronization and bit detection algorithms yield a lower mean square estimation error than traditional carrier synchronization techniques, and a small bit error rate degradation as compared to a perfectly synchronized receiver.**

I. INTRODUCTION

The problem of estimating carrier synchronization parameters is present in all digital bandpass communication systems and a wide variety of algorithms has been developed over the last 50 years to perform this task. However, over the last decade several highly-effective channel codes have been developed, such as turbo codes, low-density parity-check (LDPC) codes and bit-interleaved coded modulation (BICM) schemes. The associated iterative bit detection processes enable reliable communication at a very low signal-to-noise ratio (SNR). As a consequence, carrier synchronization must now be performed at lower SNRs than ever before. Of course, this also sets high requirements as to the synchronizers design. The fact that traditional carrier synchronization techniques failed to cope with such high-noise environments has stimulated international research into more effective carrier synchronization structures. The presented research is also to be situated in this context.

Our main contribution has been to put forward a theoretical framework for iterative synchronization structures that exchange information between the decoder and the synchronization parameter estimator [1]. From this work, we have derived a new feedforward (FF) carrier synchronization algorithm with a good performance complexity trade-off [1]. This new algorithm is observed to outperform the conventional algorithms at the normal operating SNR of coded systems provided that the carrier parameters can be considered constant over the observation interval. We have also designed and analyzed a feedback (FB) version of this algorithm [2]. The resulting FB algorithm can tolerate carrier phase variations that its FF variant is unable to cope with.

II. SYSTEM MODEL

Let us consider the transmission of the linearly modulated signal, obtained by applying a symbol vector $\mathbf{a}=\{a_k\}$ (with $E\left[|a_k|^2\right]=1$) to a square-root Nyquist transmit filter, over a bandlimited additive white Gaussian noise (AWGN) channel. Let us further assume that $\mathbf{a}$ results from the encoding and mapping of an information bit sequence $\mathbf{b}=\{b_k\}$, according to the following bit-to-symbol transition rule: $\mathbf{a}=\chi(\mathbf{b})$.

Applying the received signal to a matched filter, sampling at the optimal time instants kT (with T denoting the symbol period), and scaling by a factor $1/\sqrt{E_s}$ (with E_s denoting the average symbol energy), yields the observation vector $\mathbf{r}=\{r_k\}$ with

$$r_k = a_k e^{j\theta_k} + w_k \tag{1}$$

In (1), θ_k stands for the unknown instantaneous phase shift of the received signal vis-à-vis the local reference carrier. The trajectory of the phase shift $\{\theta_k\}$ depends upon a set of parameters referred to as carrier synchronization parameters and denoted by $\mathbf{u}$. The sequence $\{w_k\}$ consists of independent and identically distributed (i.i.d.) zero-mean complex-valued AWGN terms with independent real and imaginary parts, each having a variance equal to $N_0/2E_s$.

III. SYNCHRONIZED MAP BIT DETECTION

The ultimate task of the receiver is to recover the original information bits from the noisy observation vector $\mathbf{r}$. If a bit is detected erroneously, i.e., $\hat{b}_k \neq b_k$, a detection error occurs. Optimal detection, which minimizes the detection error for all bits, is achieved by maximum a posteriori (MAP) bit-by-bit decision. The corresponding MAP detector maximizes the a posteriori probability (APP) $p(b_k|\mathbf{r})$ with respect to b_k. In

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principle, the required marginal bit APPs $p(b_k|\mathbf{r})$ can be obtained as marginals of the joint bit and carrier synchronization parameter APP $p(\mathbf{b},\mathbf{u}|\mathbf{r})$ of all $\mathbf{b}$ and $\mathbf{u}$, which in turn can be computed from the joint likelihood function $p(\mathbf{r}|\mathbf{a},\mathbf{u})$ of $\mathbf{a}$ and $\mathbf{u}$, the a priori distributions $p(\mathbf{a})$ and $p(\mathbf{u})$ of $\mathbf{a}$ and $\mathbf{u}$, the transition rule $\mathbf{a}=\chi(\mathbf{b})$ and Bayes rule (i.e., $p(X|Y)p(Y)=p(Y|X)p(X)$). Unfortunately, the computational complexity of this procedure is commonly too high.

An elegant solution to this problem is to employ a two-stage receiver structure. First, an estimate $\hat{\mathbf{u}}$ of the carrier synchronization parameter vector $\mathbf{u}$ is derived from the observation vector $\mathbf{r}$. Then, the information bits are detected assuming ideal knowledge of the nuisance parameters, i.e., $\hat{\mathbf{u}}\equiv\mathbf{u}$. This is referred to as synchronized detection. In many practical scenarios, the (synchronized) marginal bit APPs $p(b_k|\mathbf{r},\mathbf{u}=\hat{\mathbf{u}})$ can be computed efficiently, either exactly or approximately, by applying the sum-product (SP) algorithm to a factor graph (FG) representing a suitable factorization of the (synchronized) joint APP $p(\mathbf{b},\mathbf{a}|\mathbf{r},\mathbf{u}=\hat{\mathbf{u}})$. More details on this procedure are outlined in [3]. Practical examples are the advanced detection/decoding algorithms for turbo codes, LDPC codes and BICM schemes.

IV. ML Carrier Synchronization Parameter Estimation in the Presence of Unknown Symbols

Estimation accuracy is usually measured by the mean square error (MSE) between the estimated and the true parameter values. The estimate of $\mathbf{u}$ which minimizes the MSE of all components of $\mathbf{u}$ is referred to as the minimum mean square error (MMSE) estimate. In general, however, the computation of the MMSE estimate is rather involved, and, one instead looks for the MAP estimate that maximizes the APP $p(\mathbf{u}|\mathbf{r})$ of $\mathbf{u}$. In many cases of practical interest the MAP and MMSE estimates turn out to be equal [4].

For a uniform a priori probability density function $p(\mathbf{u})$ the MAP estimation rule further reduces to the maximum likelihood (ML) estimation rule, which maximizes the likelihood function $p(\mathbf{r}|\mathbf{u})$ of $\mathbf{u}$. ML estimation is also commonly used when the a priori information $p(\mathbf{u})$ is missing.

It follows directly from (1) that the joint conditional probability density function $p(\mathbf{r}|\mathbf{a},\mathbf{u})$ of $\mathbf{r}$, given $\mathbf{a}$ and $\mathbf{u}$, is (within an irrelevant factor not depending on $\mathbf{u}$) given by

$$p(\mathbf{r}|\mathbf{a},\mathbf{u})\propto\exp\left(2\frac{E_s}{N_0}\mathrm{Re}\left\{\sum_k a_k^* z_k(\mathbf{u})\right\}\right) \tag{2}$$

with

$$z_k(\mathbf{u})=r_k e^{-j\theta_k} \tag{3}$$

Evaluating the likelihood function $p(\mathbf{u}|\mathbf{r})$ requires averaging $p(\mathbf{r}|\mathbf{a},\mathbf{u})$ from (2) over all possible values of the symbol vector $\mathbf{a}$, which gives rise to a computational burden that is exponential in the number of symbols. Hence, true ML estimation cannot be used for large blocks, in which case one has to resort to further approximation techniques.

One of these techniques is the expectation-maximization (EM) algorithm [5]. Considering $\mathbf{r}$ as the incomplete observation and $(\mathbf{r},\mathbf{a})$ as the complete observation, the EM algorithm states that the sequence $\hat{\mathbf{u}}^{(i)}$ defined by

$$\hat{\mathbf{u}}^{(i)}=\arg\max_{\mathbf{u}} Q\left(\mathbf{u},\hat{\mathbf{u}}^{(i-1)}\right) \tag{4}$$

with

$$Q\left(\mathbf{u},\hat{\mathbf{u}}^{(i-1)}\right)=E_{\mathbf{a}}\left[\ln p(\mathbf{r},\mathbf{a}|\mathbf{u})\middle|\mathbf{r},\hat{\mathbf{u}}^{(i-1)}\right] \tag{5}$$

converges to the ML estimate provided that the initial estimate $\hat{\mathbf{u}}^{(0)}$ is sufficiently close to the global maximum of the log-likelihood function $\ln p(\mathbf{r}|\mathbf{u})$ of $\mathbf{u}$; otherwise, convergence to a different stationary point may occur. By using Bayes rule and considering that the distribution of $\mathbf{a}$ does not depend on $\mathbf{u}$,

$$p(\mathbf{r},\mathbf{a}|\mathbf{u})=p(\mathbf{r}|\mathbf{a},\mathbf{u})p(\mathbf{a}|\mathbf{u})=p(\mathbf{r}|\mathbf{a},\mathbf{u})p(\mathbf{a}) \tag{6}$$

Therefore, substituting (6) in (5), we get

$$Q\left(\mathbf{u},\hat{\mathbf{u}}^{(i-1)}\right)=\sum_{\mathbf{a}} p\left(\mathbf{a}\middle|\mathbf{r},\hat{\mathbf{u}}^{(i-1)}\right)\ln p(\mathbf{r}|\mathbf{a},\mathbf{u})+\sum_{\mathbf{a}} p\left(\mathbf{a}\middle|\mathbf{r},\hat{\mathbf{u}}^{(i-1)}\right)\ln p(\mathbf{a}) \tag{7}$$

The second term in (7) does not depend on $\mathbf{u}$, and as far as the M-step is concerned, it can be dropped. Consequently, it follows from (2) that

$$Q\left(\mathbf{u},\hat{\mathbf{u}}^{(i)}\right)\propto 2\frac{E_s}{N_0}\mathrm{Re}\left\{\sum_k \mu_k^*\left(\hat{\mathbf{u}}^{(i)}\right)z_k(\mathbf{u})\right\} \tag{8}$$

where

$$\mu_k\left(\hat{\mathbf{u}}^{(i)}\right)=\sum_{\mathbf{a}} p\left(\mathbf{a}\middle|\mathbf{r},\hat{\mathbf{u}}^{(i)}\right)a_k=\sum_{a_k} p\left(a_k\middle|\mathbf{r},\hat{\mathbf{u}}^{(i)}\right)a_k \tag{9}$$

denotes the a posteriori expectation of the k th symbol a_k conditioned on $\mathbf{r}$ and $\mathbf{u}$.

V. ML Carrier Parameter Estimation and MAP Bit Detection

We emphasize that no approximation is involved in obtaining (9); the right hand side simply expresses the a posteriori average of a_k in terms of the marginal APP $p\left(a_k\middle|\mathbf{r},\hat{\mathbf{u}}^{(i)}\right)$ of a_k, rather than the joint APP $p\left(\mathbf{a}\middle|\mathbf{r},\hat{\mathbf{u}}^{(i)}\right)$ of all components of $\mathbf{a}$. In other words, due to the particular structure of the digital data-modulated signal, the estimation of the carrier synchronization parameter vector $\mathbf{u}$ via the EM algorithm only requires the knowledge of *marginal* symbol APPs $p\left(a_k\middle|\mathbf{r},\hat{\mathbf{u}}^{(i)}\right)$. This makes ML synchronizers operating according to the EM algorithm and MAP bit detectors operating according to the SP algorithm on a FG of $p\left(\mathbf{b},\mathbf{a}\middle|\mathbf{r},\mathbf{u}=\hat{\mathbf{u}}^{(i)}\right)$ complementary, since the marginal probabilities needed by the first one are almost automatically provided by the second one (as by-products). This is because the SP algorithm typically computes *all* the marginals of a function in parallel.

A major drawback of this estimator/detector combination is its computational overhead: for each estimator iteration the marginal APPs for all data symbols need to be computed. It is well-known that when the FG of $p(\mathbf{b},\mathbf{a}|\mathbf{r},\mathbf{u}=\hat{\mathbf{u}}^{(i)})$ contains cycles, the computation of these APPs according to the SP algorithm is itself an iterative process (as happens with turbo codes, LDPC codes and BICM schemes). Hence, in principle, for each estimator iteration we should reset to zero the state information in the FG and subsequently perform enough SP iterations to achieve convergence of the APPs.

To reduce the computational complexity, we propose the following approximate implementation: after each update of $\hat{\mathbf{u}}^{(i)}$ we perform only a single SP iteration, but maintain state information within the FG from one EM iteration to the next. Hence, estimator and detector iterations are intertwined, which drastically reduces the computational overhead related to the iterative estimation process (4),(8).

Note however that, strictly speaking, the simplified implementation is no longer an EM algorithm. Indeed, performing only one SP iteration at each EM iteration may lead to poor approximations of marginal symbol APPs, especially for the first EM iterations. Moreover, non-resetting the FG state information implies that the information computed by the SP algorithm will be a function of all the previous estimates $\hat{\mathbf{u}}^{(0)}$, $\hat{\mathbf{u}}^{(1)}$, …, $\hat{\mathbf{u}}^{(i-1)}$ and not only of $\hat{\mathbf{u}}^{(i-1)}$ as required by the EM algorithm.

The approximate implementation actually corresponds to the one introduced in an ad hoc fashion by Lottici [6] for the particular case of a turbo-coded scheme. We have provided here a mathematical interpretation of Lotticis' algorithm by showing that it may actually be regarded as an EM algorithm approximation. More generally, the idea of using the available information provided by a soft-information-based receiver to help synchronization has been applied in a large number of contributions (see references in [1]) and the EM approach was one of the first attempts to provide a theoretical framework for this category of (ad hoc) algorithms. Since then, other frameworks based on gradient methods [7] or the sum-product algorithm [3] have been proposed in the literature.

VI. ML-Based Feedforward Phase and Frequency Estimation

A. Algorithm Derivation

The EM algorithm (4),(8) directly estimates the carrier synchronization parameter vector $\mathbf{u}$ from the matched filter output samples, before a correction has been applied to them. For this reason, it is referred to as FF algorithm. The main drawback of a FF algorithm is that, since it derives only one estimate of $\mathbf{u}$ per observation interval, its operation is mainly restricted to the estimation of carrier synchronization parameters that remain essentially constant over that period of time.

A common example is the case where the instantaneous phase shift is linear in time: $\theta_k = \theta + 2\pi FkT$, i.e., characterized by a constant frequency offset F and a constant reference phase shift θ at $k=0$. The value of F and θ is assumed to be uniformly distributed in the intervals $[-F_{\max}, F_{\max}]$ and $[-\pi,\pi]$, respectively. Defining the carrier synchronization parameter vector $\mathbf{u} = (F\theta)^T$, the resulting EM estimator is

$$\hat{F}^{(i)} = \arg\max_{\tilde{F}}\left(\left|\sum_k \mu_k^*\left(\hat{F}^{(i-1)},\hat{\theta}^{(i-1)}\right)r_k e^{-j2\pi k\tilde{F}T}\right|\right) \tag{10}$$

$$\hat{\theta}^{(i)} = \arg\left\{\sum_{k\in} \mu_k^*\left(\hat{F}^{(i-1)},\hat{\theta}^{(i-1)}\right)r_k e^{-j2\pi k\hat{F}^{(i)}T}\right\} \tag{11}$$

B. Simulation Results

We now resort to simulation to derive actual performance results (in terms of MSE and bit error rate (BER)) of a system that combines iterative EM carrier synchronization with iterative MAP bit detection. We take 4-PSK as the signal constellation, and choose for the channel code a memory order 4 turbo code. The turbo encoder encompasses the parallel concatenation of two identical binary 16-state rate-1/2 recursive systematic convolutional encoders with generator polynomials $(21)_8$ and $(37)_8$ in octal notation, via a pseudo random interleaver with block length 900 information bits, and an appropriate puncturing pattern so that the block at the turbo encoder output comprises 1800 coded bits. This binary turbo code is followed by conventional Gray-mapped 4-PSK modulation, giving rise to a block of 900 random data symbols. Finally, 100 pilot symbols are added, yielding a transmitted burst of 1000 symbols. We further assume that the pilot symbols are organized into two blocks of 50 consecutive pilot symbols with 100 data symbols in between, and symmetrically positioned about the center of this burst, which in turn corresponds to k=0. Simulations are performed until at least 10000 bursts are transmitted, and at least 100 frame errors are counted. For each burst a new reference phase shift and a new frequency offset are taken from a random uniform distribution over $[-0.1/T, 0.1/T]$ and $[-\pi,\pi]$, respectively. The reference phase shift estimation error is measured modulo 2π and supported in the interval $[-\pi,\pi]$. The initial estimate required to start the iterative joint synchronization/detection process is obtained from either the conventional DA or the conventional two-stage DA/4th-power method (see [8]). We will further refer to these schemes as 'DA init.' and 'DA/4th-power init.', respectively.

Results not shown here indicate that, in the case of perfect synchronization, the conventional turbo decoder needs about 9 iterations to converge, and has a normal operating SNR (say, corresponding to a BER of less than 10^{-3}) of E_b/N_0 larger than 1.75 dB, or equivalently, E_s/N_0 larger than 1.3 dB.

Fig. 1 shows the MSE pertaining to the EM estimation of F (similar results not reported here have been obtained for θ) after 10 joint synchronizer/detector iterations. For the sake of comparison the MSE resulting from the conventional DA and the two-stage DA/4th-power methods is also displayed. The advantage of the proposed algorithm over the conventional methods is clear: at the normal operating SNR of the code the proposed algorithm significantly outperforms the conventional ones. The results also indicate the importance of an accurate initial estimate. The estimator with two-stage DA/4th-power initialization provides a considerable improvement over the estimator with DA initialization within the useful SNR range of the turbo code.

Fig. 2 shows the BER after 10 decoding iterations of the different system configurations under consideration. It is observed that the system with conventional DA synchronization exhibits a very large BER degradation as compared to the case of perfect synchronization, and that the system with conventional DA/4th-power synchronization gives rise to a BER degradation of 0.25 dB at 10^{-3}. However, this already small BER degradation may be further reduced to about 0.1 dB, by performing joint synchronization/decoding iterations (provided that DA/4th-power initialization is used). When less accurate DA initialization is employed, using the proposed joint detection and synchronization algorithm instead of a conventional DA/4th-power synchronizer significantly degrades the BER performance, although it substantially reduces the MSE at the normal operating SNR of the detector. This clearly indicates the importance of a sufficiently accurate initial estimate to ensure the *joint* convergence of the intertwined synchronization and detection processes (EM algorithm approximation).

VII. ML-Based FeedBack Phase Tracking

A. Algorithm Derivation

So far we have assumed that the instantaneous phase shift θ_k is linear in time. In practice, however, θ_k is always subject to random fluctuations due to imperfections in the transmitter and receiver oscillators.

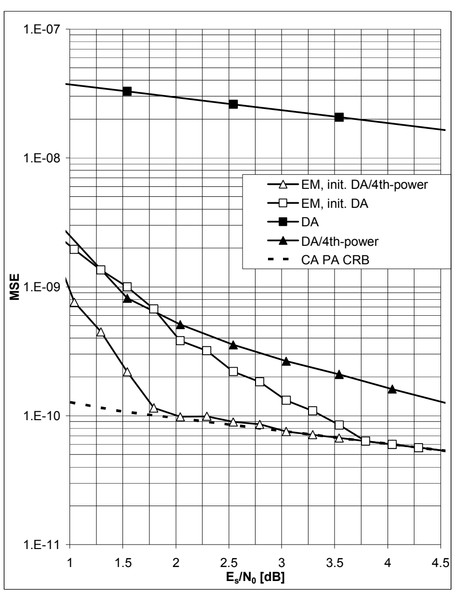

Fig. 1: MSE resulting from frequency offset estimation

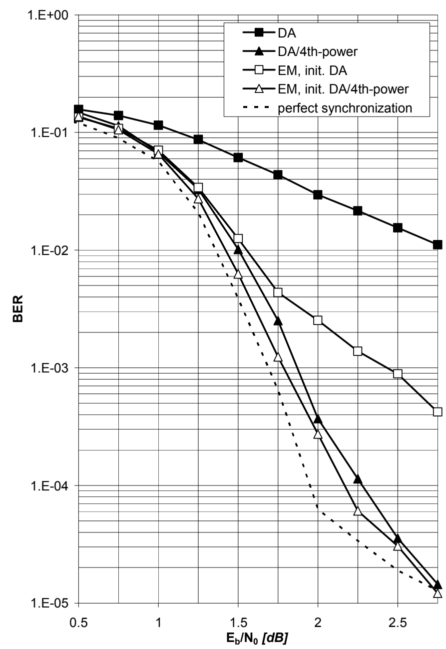

Fig. 2: BER resulting from turbo receiver with perfect synchronization, conventional DA or two-stage DA/4th-power synchronization, or iterative joint synchronization/detection.

That is

$$\theta_k = \theta + 2\pi FkT + n_k \tag{12}$$

where n_k stands for a zero-mean stochastic random process, referred to as phase noise. An often used phase noise model is based on a discrete Wiener process (random walk):

$$n_k = n_{k-1} + v_{k-1} \tag{13}$$

characterized by i.i.d. Gaussian increments v_{k-1} with zero mean and root mean square (rms) value σ_v, descriptive of the phase noise intensity. Note that the variance of the phase noise increases linearly with the absolute value of the symbol index k: $E\left[(n_k)^2\right] = |k|(\sigma_v)^2$. Hence, for long bursts (i.e., the common situation for codes with large coding gains), the fluctuation of the phase noise over the burst cannot be ignored.

A possible solution to this problem is the use of a FB algorithm. As opposed to FF synchronizers, FB synchronizers derive from the phase-corrected matched filter output samples an estimate of the instantaneous phase error $\hat{\theta}_k - \theta_k$, and feed this estimate back to the phase correction block. As a result, FB structures inherently have the ability to automatically track slow variations of the instantaneous phase shift.

It is well known that FB schemes can be based upon FF algorithms [9]. Inspired by the EM algorithm, the following *iterative* second order PLL is proposed. During the i th iteration, the phase and frequency estimates are updated according to the following recursion

$$\begin{pmatrix} \hat{\theta}_{k+1}^{(i)} \\ 2\pi\hat{F}_{k+1}^{(i)}T \end{pmatrix} = \begin{pmatrix} \hat{\theta}_k^{(i)} + 2\pi\hat{F}_k^{(i)}T + ax_k^{(i)} \\ 2\pi\hat{F}_k^{(i)}T + bx_k^{(i)} \end{pmatrix} \tag{14}$$

where a and b are loop parameters that control the equivalent noise bandwidth and damping factor. The phase error detector (PED) output $x_k^{(i)}\left(\hat{\theta}_k^{(i)}-\theta_k\right)$ is given by

$$x_k^{(i)}\left(\hat{\theta}_k^{(i)}-\theta\right)=\frac{d}{d\theta_k}Q\left(\mathbf{u},\hat{\mathbf{u}}^{(i-1)}\right)=\mathrm{Im}\left\{\mu_k^*\left(\hat{\mathbf{u}}^{(i-1)}\right)r_k e^{-j\hat{\theta}_k^{(i)}}\right\} \qquad (15)$$

with the carrier synchronization parameter vector now defined as $\mathbf{u}\equiv\{\theta_k\}$. An initial estimate vector $\hat{\mathbf{u}}^{(0)}$ is needed to start the iterations. This may be obtained by performing a conventional non-code-aided FB recursion. More details on the operation of the proposed FB synchronizer can be found in [10].

B. Simulation Results

We consider the turbo coded transmission scheme from Section VI.B, but with an interleaver of size 3333 bits, no interleaving and 2-PSK mapping. As in the FF case, the synchronizer iterations are merged with the turbo-decoder iterations to reduce the complexity. A total of 10 joint detector/synchronizer iterations are carried out. The loop parameters from (14) are selected such that the loop bandwidth equals 0.0075 and the damping factor equals 0.707, when the quantities $\mu_k\left(\hat{\mathbf{u}}^{(i)}\right)$ would equal the actual symbols. This choice of the loop bandwidth is a trade-off: small enough to reduce the effect of AWGN on the phase error, but large enough to track phase noise and to limit the acquisition time caused by the frequency error at the first iteration. We take for $\hat{\theta}_0^{(0)}$ a phase estimate obtained from a sequence of 32 pilot symbols preceding the actual data, and set $\hat{F}_0^{(0)}=0$. A block of 16 pilot symbols inserted into the symbol stream once every 256 coded symbols enables to check and compensate for cycle slips. The performance of the FB synchronizer is assessed in terms of BER versus E_b/N_0.

Fig. 3 also shows the BER for a perfectly synchronized system, together with the BER as obtained with the FF phase and frequency estimator operating on the same signal and with identical initialization. We make the following observations:

- When the instantaneous phase shift is constant, the receiver with FF synchronization yields essentially the same BER as the perfectly synchronized receiver, and outperforms the receiver with FB synchronization by about 0.2 dB in the waterfall region of the turbo code. The BER degradation of the latter receiver can be made vanishingly small by reducing the loop bandwidth, because the carrier phase is time-invariant.
- The BER performance of the receiver with FF synchronization is considerably degraded in the presence of a moderately time-varying instantaneous phase shift. The receiver with FB synchronization can handle phase shift variations that are too large for proper operation of the receiver with FF synchronization. As $(\mathrm{FT},\sigma_v)=(10^{-3},0)$ and $(\mathrm{FT},\sigma_v)=(0,0)$ yield the same BER performance, it follows that the BER degradation as compared to perfect synchronization is caused by the additive-noise component of the phase error. A slightly larger degradation occurs for $(\mathrm{FT},\sigma_v)=(0,1.08^{\circ})$, because a phase-noise component is added to the phase error.

REFERENCES

[1] N. Noels, V. Lottici, A. Dejonghe, H. Steendam, M. Moeneclaey, M. Luise and L. Vandendorpe, “A Theoretical Framework for Soft Information Based Synchronization in Iterative (Turbo) Receivers,” *EURASIP J. on Wireless Commun. and Networking*, vol. 2005, no. 2, April 2005, pp. 117-129

[2] N. Noels, H. Steendam and M. Moeneclaey "Carrier Phase Tracking From Turbo and LDPC Coded Signals Affected By a Frequency Offset" , *IEEE Communications Letters*, vol. 9, no. 10, Oct. 2005, pp. 915-917

[3] F. Kschinschang, B. Frey and H.-A. Loeliger, “Factor Graphs and the sum-product algorithm,” *IEEE Trans. Inform. Theory*, vol. 47, no. 2, Feb. 2001, pp. 498-519

[4] H. L Van Trees, “*Detection, Estimation, and Modulation Theory*,” New York, Wiley, 1990

[5] T. Moon, “The expectation-maximization algorithm,” *IEEE Signal Processing Magazine*, vol. 13, No. 6, June 1996, 99. 47-60

[6] V. Lottici and M. Luise, “Embedding carrier phase recovery into iterative decoding of turbo-coded linear modulations,” *IEEE Transactions on Communications*, vol. 52, no. 4, April 2004, pp. 661-669

[7] D.P. Bertsekas, “*Nonlinear Programming*,” Athena Scientific, USA, 2003

[8] N. Noels, H. Steendam, M. Moeneclaey, H. Bruneel, "Carrier Phase and Frequency Estimation for Pilot-Symbol Assisted Transmission: Bounds and Algorithms" , *IEEE Transactions on Signal Processing*, vol. 53, no. 12, Dec. 2005, pp. 4578-4587

[9] H. Meyr, M. Moeneclaey and S. A. Fechtel, “Digital Communication Receivers, Synchronization, Channel Estimation and Signal Processing,” New York, Wiley, 1998

[10] N. Noels, H. Steendam and M. Moeneclaey "Performance Analysis of ML-Based Feedback Carrier Phase Synchronizers for Coded Signals,” *IEEE Transactions on Signal Processing*, vol. 55, no. 3, March 2007, pp. 1129-1136

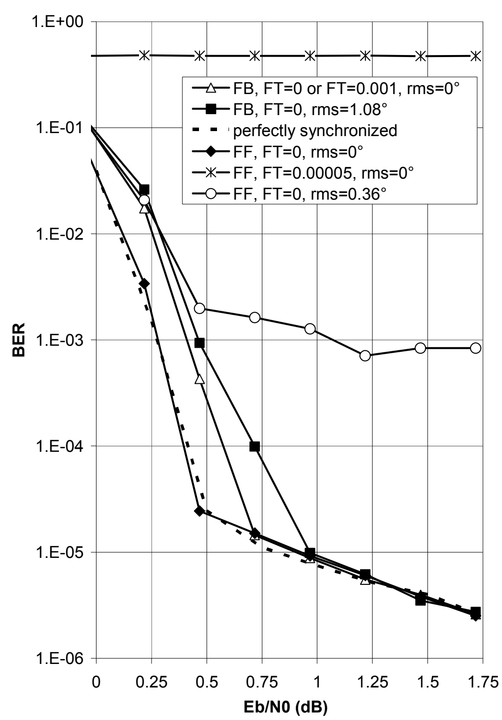

Fig. 3: BER performance of turbo code with FF or FB carrier synchronization