GHENT UNIVERSITY

ARTIFICIAL INTELLIGENCE (E016350)
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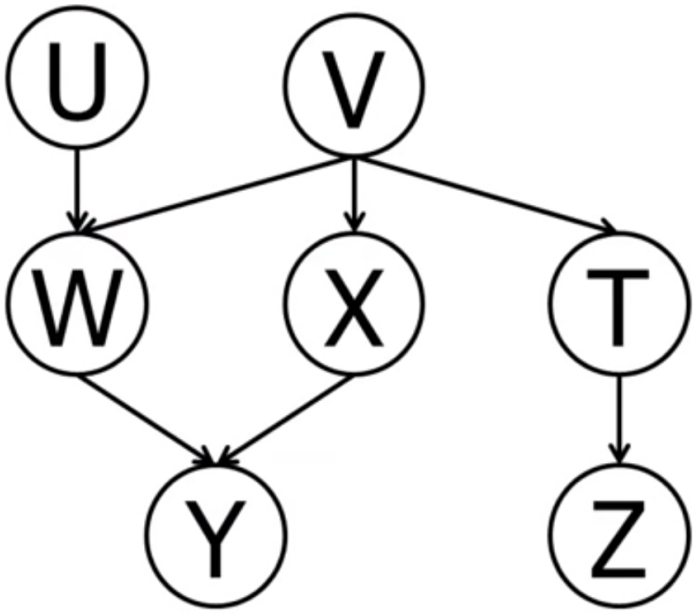
Exercises: Bayesian networks

1. Consider the Bayes' net given below. Remember that $X \perp\!\!\!\perp Y$ reads as "X is independent of Y given nothing", and $X \perp\!\!\!\perp Y \mid \{Z, W\}$ reads as "X is independent of Y given Z and W". For each expression, indicate whether it is guaranteed to be true or not.



1) $V \perp\!\!\!\perp Z$
2) $V \perp\!\!\!\perp Z \mid T$
3) $U \perp\!\!\!\perp V$
4) $U \perp\!\!\!\perp V \mid W$
5) $U \perp\!\!\!\perp V \mid X$
6) $U \perp\!\!\!\perp V \mid Y$
7) $U \perp\!\!\!\perp V \mid Z$
8) $W \perp\!\!\!\perp X$
9) $X \perp\!\!\!\perp T \mid V$
10) $X \perp\!\!\!\perp W \mid U$
11) $Y \perp\!\!\!\perp Z$
12) $Y \perp\!\!\!\perp Z \mid T$
13) $Y \perp\!\!\!\perp Z \mid X$
14) $Y \perp\!\!\!\perp Z \mid V$
15) $W \perp\!\!\!\perp Z \mid V$
16) $U \perp\!\!\!\perp Z$
17) $U \perp\!\!\!\perp Z \mid Y$
18) $U \perp\!\!\!\perp Z \mid \{V, Y\}$

2. Consider Bayesian networks from Figure below, where $S = Smoking$, $L = LungCancer$, $C = Cough$, $B = BiopsyTest$ ($BiopsyTest$ is positive only if the result of the test is positive for cancer). All variables are Boolean and the test population consists of 60-years old people who are not smokers or who have smoked for the last 40 years.

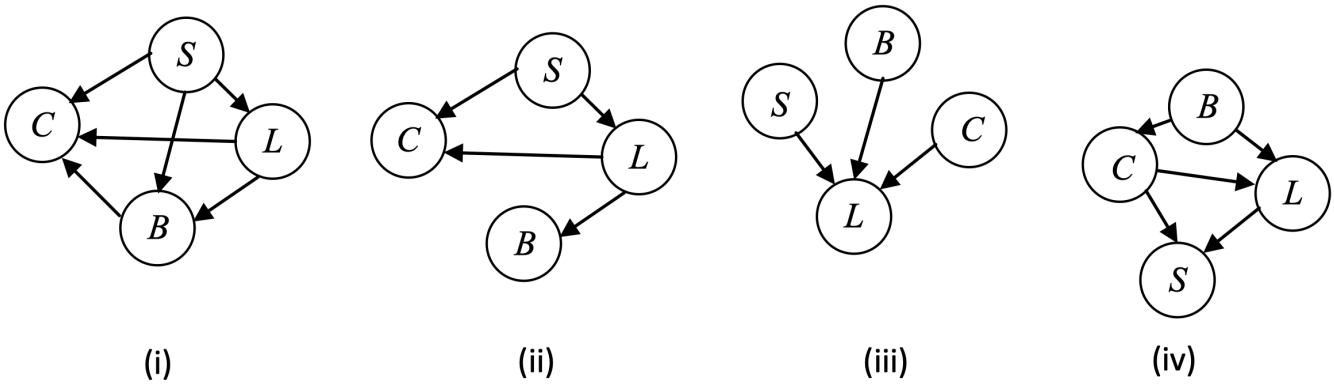


Figure 1: Examples of Bayesian networks.

(a) Which networks are correct, based on the common knowledge about this disease?

(b) Which network has the least amount of parameters? Why?

(c) Write reasonable values for conditional probabilities (CPT) for the node C in the network (ii).

(d) Using the network (ii), derive a symbolic expression for $\mathrm{P}(B \mid S)$ in terms of conditional probabilities that should be available in the CPT tables (do not use any concrete values for the entries in the CPT tables).

(e) Make a similar derivation for $\mathrm{P}(L \mid B)$, also for the network (ii).

3. Suppose that a patient can have a symptom (S) that can be caused by two different diseases (A and B). It is known that the variation of gene G plays a big role in the manifestation of disease A. The Bayes' Net and corresponding conditional probability tables for this situation are shown below. For each part, you may leave your answer as an arithmetic expression.

(a) Compute the following entry from the joint distribution:
$\mathrm{P}(g, a, b, s) =$

(b) What is the probability that a patient has disease A?
$\mathrm{P}(a) =$

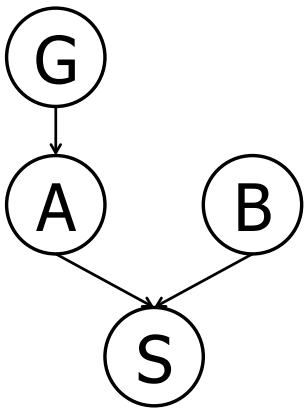
(c) What is the probability that a patient has disease A given that they have disease B?
$\mathrm{P}(a \mid b) =$

(d) What is the probability that a patient has disease A given that they have symptom S and disease B?
$\mathrm{P}(a \mid s, b) =$

(e) What is the probability that a patient has the disease carrying gene variation G given that they have disease A?
$\mathrm{P}(g \mid a) =$

$\mathbf{P}(G)$	
g	0.1
$\neg g$	0.9

$\mathbf{P}(A \mid G)$		
a	g	1.0
a	$\neg g$	0.1
$\neg a$	g	0.0
$\neg a$	$\neg g$	0.9



$\mathbf{P}(B)$	
b	0.4
$\neg b$	0.6

$\mathbf{P}(S \mid A, B)$			
s	a	b	1.0
s	a	$\neg b$	0.9
s	$\neg a$	b	0.8
s	$\neg a$	$\neg b$	0.1
$\neg s$	a	b	0.0
$\neg s$	a	$\neg b$	0.1
$\neg s$	$\neg a$	b	0.2
$\neg s$	$\neg a$	$\neg b$	0.9

(f) What is the probability that a patient has the disease carrying gene variation G given that they have disease B?
$\mathrm{P}(g \mid b) =$