ARTIFICIAL INTELLIGENCE (E016350B)
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Exercises: Inference in Bayesian networks

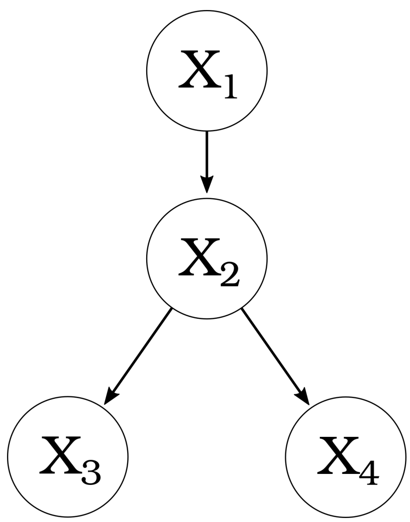
1. Given the Bayesian network below, calculate marginal and conditional probabilities $\mathrm{P}(\neg x_3)$ and $\mathrm{P}(x_2 \mid \neg x_3)$ by using:

   (a) the method of **inference by enumeration**,

   (b) the method of **variable elimination**.

$\mathbf{P}(X_1)$	
x_1	0.4
$\neg x_1$	0.6

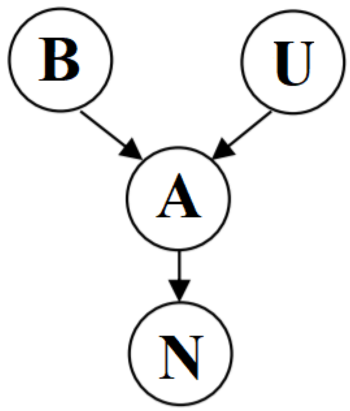
$\mathbf{P}(X_3 \mid x_2)$		
x_2	x_3	0.2
x_2	$\neg x_3$	0.8
$\neg x_2$	x_3	0.3
$\neg x_2$	$\neg x_3$	0.7



$\mathbf{P}(X_2 \mid X_1)$		
x_1	x_2	0.8
x_1	$\neg x_2$	0.2
$\neg x_1$	x_2	0.5
$\neg x_1$	$\neg x_2$	0.5

$\mathbf{P}(X_4 \mid X_2)$		
x_2	x_4	0.8
x_2	$\neg x_4$	0.2
$\neg x_2$	x_4	0.5
$\neg x_2$	$\neg x_4$	0.5

2. Suppose a burglary alarm which can function in its basic mode and with an additional ultrasonic sensor (which can be turned on or off). The network in Figure 2 represents a complete burglar alarm and notification system, with the following binary random variables: B (burglary happens when $B = 1$), U (ultrasonic sensor is on when $U = 1$), A (alarm sounds when $A = 1$) and N (neighbor calls when $N = 1$).



When the ultrasonic sensor is active, the probability that alarm is properly activated by burglary is increased from x to y, but the probability of false alarm is also increased, from ε to 2ε. (When the ultrasonic sensor is not active, the probability of false alarm is ε, and with ultrasonic sensor on, the false alarm appears with probability 2ε). Prior probability of burglary in the neighborhood where the house is located is b and the probability that the ultrasonic sensor is active is u.

(a) Write the joint probability for the network in Fig. 2 (a). Write also a table, which displays conditional probabilities of A given B and U.

(b) Express the probability that there is a burglary if alarm goes on in terms of x, y, u, ε and b.

(c) Suppose that the neighbor calls to report the alarm with probability n when the alarm is on and never when the alarm is off. Express the probability that there is a burglary if the neighbor calls and if the ultrasonic sensor is off.

(d) Extend the network from Figure 2(a) so that it can represent the following statements:

(i) the alarm can be activated by pets;

(ii) when the alarm goes on, an SMS notification is sent automatically via the Internet.

3. PacLabs has just created a new type of mini power pellet that is small enough for Pacman to carry around with him when he's running around mazes. Unfortunately, these mini-pellets don't guarantee that Pacman will win all his fights with ghosts, and they look just like the regular dots Pacman carried around to snack on.

   Pacman just ate a snack (P), which was either a mini-pellet (p), or a regular dot ($\neg p$), and is about to get into a fight (W), which he can win (w) or lose ($\neg w$). Both these variables are unknown, but fortunately, Pacman is a master of probability. He knows that his bag of snacks has 5 mini-pellets and 15 regular dots. He also knows that if he ate a mini-pellet, he has a 70% chance of winning, but if he ate a regular dot, he only has a 20% chance.

   (a) What is P(w), the marginal probability that Pacman will win?

   (b) Pacman won! Hooray! What is the conditional probability P($p \mid w$) that the food he ate was a mini-pellet, given that he won?

   (c) Pacman can make better probability estimates if he takes more information into account. First, Pacman's breath, B, can be bad (b) or fresh ($\neg b$). Second, there are two types of ghost (M): mean (m) and nice ($\neg m$). Pacman has encoded his knowledge about the situation in the following probability distribution

$$\mathbf{P}(M, P, B, W) = \mathbf{P}(M)\mathbf{P}(P)\mathbf{P}(W \mid M, P)\mathbf{P}(B \mid P).$$

   Based on the given probability distribution, in the box below draw the corresponding Bayesian network.

$\mathbf{P}(M)$	
m	0.5
$\neg m$	0.5

$\mathbf{P}(W \mid M, P)$			
m	p	w	0.60
m	p	$\neg w$	0.40
m	$\neg p$	w	0.10
m	$\neg p$	$\neg w$	0.90
$\neg m$	p	w	0.80
$\neg m$	p	$\neg w$	0.20
$\neg m$	$\neg p$	w	0.30
$\neg m$	$\neg p$	$\neg w$	0.70

$\mathbf{P}(P)$	
p	0.25
$\neg p$	0.75

$\mathbf{P}(B \mid P)$		
p	b	0.80
p	$\neg b$	0.20
$\neg p$	b	0.40
$\neg p$	$\neg b$	0.60

   (d) Just based on the structure, which of the following are guaranteed to be true? Explain your answers.

   (1) $W \perp\!\!\!\perp B$

   (2) $W \perp\!\!\!\perp B \mid P$

   (3) $M \perp\!\!\!\perp B$

   (4) $M \perp\!\!\!\perp B \mid P$