**ARTIFICIAL INTELLIGENCE (E016350)**
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# Neural networks – Part 1

1. **[Old exam question]**

   We consider a binary classification problem where the input data are two-dimensional. The table below shows the first five examples in the training set.

| Example | $x_1$ | $x_2$ | Class |
|---|---|---|---|
| 1 | 10 | 20 | + |
| 2 | 4 | 15 | - |
| 3 | 1 | 15 | + |
| 4 | 1 | 10 | - |
| 5 | 5 | 20 | + |

   (a) Assign to the class '+' the label $y = 1$ and to the class '-' the label $y = 0$. Initialize the weight vector as $\mathbf{w} = [1, 2, 3]^\top$, where the first entry is for the bias term. Run the perceptron learning rule on the five examples, with the learning rate of 1, and fill in the following table and show your calculation:

| Example | Score | Predicted class | Updated weights |
|---|---|---|---|
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |

(b) Now keep all the same as in (a) but use a different convention for the class labels $y$: for the class '+' use $y = 1$ and for the class '-' use $y = -1$. Will now the perceptron learning rule lead to different updated weights? Explain your answer and show the calculation for the updated weights based on the first two examples from the given dataset.

The updated weights will ◯ change ◯ not change

because

For Example 1:

updated weights: ________________________________________

For Example 2:

updated weights: ________________________________________

For Example 3:

updated weights: ________________________________________

For Example 4:

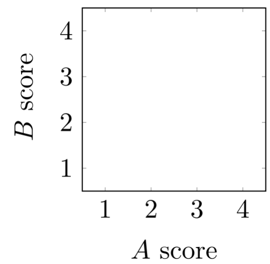
updated weights: ________________________________________

For Example 5:

updated weights: ________________________________________

2. You want to predict if movies will be profitable based on their screenplays. You hire two critics $A$ and $B$ to read a script you have and rate it on a scale of 1 to 4. The critics are not perfect; here are five data points including the critics' scores and the performance of the movie

| # | Movie name | A | B | Profit? |
|---|---|---|---|---|
| 1 | Pellet Power | 1 | 1 | - |
| 2 | Ghosts! | 3 | 2 | + |
| 3 | Pac is Bac | 2 | 4 | + |
| 4 | Not a Pizza | 3 | 4 | + |
| 5 | Endless Maze | 2 | 3 | - |




(a) First, you would like to examine the linear separability of the data. Plot the data on the 2D plane above, label profitable movies with + and non-profitable movies with - and determine if the data are linearly separable.

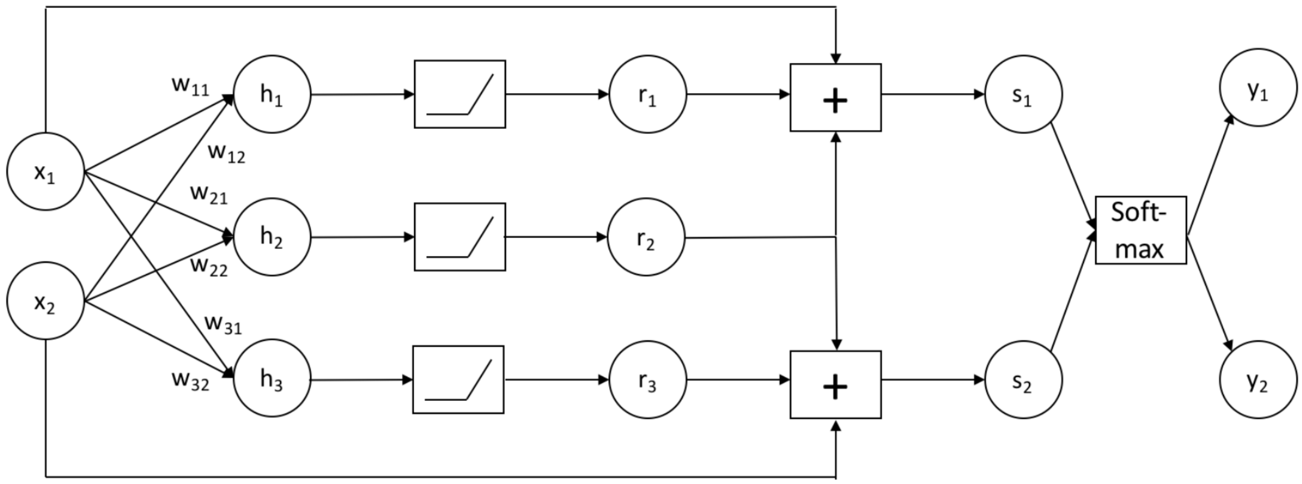
(b) Now you decide to use a single-layer perceptron to classify your data. Suppose you directly use the points given above as features. That is $f_1$ = points given by $A$ and $f_2$ = points given by $B$.

Run one pass through the data with the perceptron learning algorithm, filling out the table below. Initialize the weights as $(-1, 0, 0)$, where the first entry denotes the bias, and set the learning rate to 1. Go through the data points in the given order using data point #1 at step 1.

| Step | Current weights | Score (show your calculation here) | Updated weights |
|---|---|---|---|
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |

(c) Have weights been learned that separate the data?

3. The network below is a neural network with inputs $x_1$ and $x_2$, and outputs $y_1$ and $y_2$. The internal nodes are computed below. All variables are scalar values.
Note that $\text{ReLU}(x) = \max(0, x)$.




The expressions for the internal nodes in the network are given here for convenience:

$$h_1 = w_{11}x_1 + w_{12}x_2 \qquad h_2 = w_{21}x_1 + w_{22}x_2 \qquad h_3 = w_{31}x_1 + w_{32}x_2$$

$$r_1 = \text{ReLU}(h_1) \qquad r_2 = \text{ReLU}(h_2) \qquad r_3 = \text{ReLU}(h_3)$$

$$s_1 = x_1 + r_1 + r_2 \qquad s_2 = x_2 + r_2 + r_3$$

$$y_1 = \frac{\exp(s_1)}{\exp(s_1) + \exp(s_2)} \qquad y_2 = \frac{\exp(s_2)}{\exp(s_1) + \exp(s_2)}$$

(a) **Forward Propagation:** Suppose for this part that

$$x_1 = 3,\ x_2 = 5,\ w_{11} = -10,\ w_{12} = 7,\ w_{21} = 2,\ w_{22} = 5,\ w_{31} = 4,\ w_{32} = -4.$$

What are the values of the following internal nodes? Please simplify any fractions.

(1) $h_1 =$ ____________

(2) $s_1 =$ ____________

(3) $y_2 =$ ____________

(b) **Back Propagation:** Compute the following gradients analytically. The answer should be an expression of any of the nodes in the network $(x_1, x_2, h_1, h_2, h_3, r_1, r_2, r_3, s_1, s_2, y_1, y_2)$ or weights $w_{11}$, $w_{12}$, $w_{21}$, $w_{22}$, $w_{31}$, $w_{32}$. In the case where the gradient depends on the value of nodes in the network, please list all possible analytical expressions, caused by active/inactive ReLU, in form of a set $\{\dots\}$.

(1) $\dfrac{\partial h_2}{\partial x_1} =$ ____________ (3) $\dfrac{\partial r_3}{\partial w_{31}} =$ ____________ (5) $\dfrac{\partial s_1}{\partial x_1} =$ ____________

(2) $\dfrac{\partial h_1}{\partial w_{21}} =$ ____________ (4) $\dfrac{\partial s_1}{\partial r_1} =$ ____________ (6) $\dfrac{\partial y_2}{\partial s_2} =$ ____________

4. You are asked to classify the following datasets:

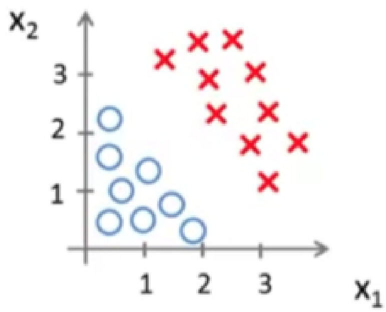



Figure 1: Dataset 1

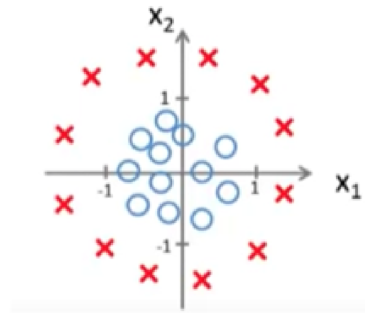



Figure 2: Dataset 2

Using the given features and the specified dataset, which model(s) can achieve a training accuracy of 1?

(i) Dataset 1: $X_1$, $X_2$

- ☐ Single-layer perceptron
- ☐ Logistic regression
- ☐ Neural network
- ◯ None of the above

(i) Dataset 2: $X_1$, $X_2$

- ☐ Single-layer perceptron
- ☐ Logistic regression
- ☐ Neural network
- ◯ None of the above

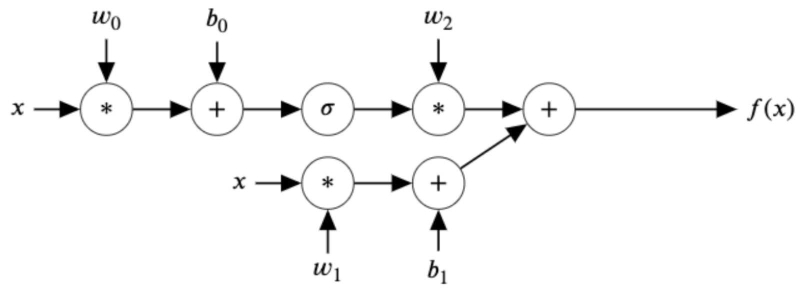
(i) Dataset 2: $X_1$, $X_2$, $X_1X_2$

- ☐ Single-layer perceptron
- ☐ Logistic regression
- ☐ Neural network
- ◯ None of the above

(i) Dataset 2: $X_1$, $X_2$, $X_1^2$, $X_2^2$

- ☐ Single-layer perceptron
- ☐ Logistic regression
- ☐ Neural network
- ◯ None of the above

5. A neural network is presented below. $\sigma(x) = \frac{1}{1+e^{-x}} = \frac{e^x}{e^x+1}$ and $tanh(x) = \frac{e^{2x}-1}{e^{2x}+1}$.




Which of the following statements are correct about this neural network? Mark all that apply.

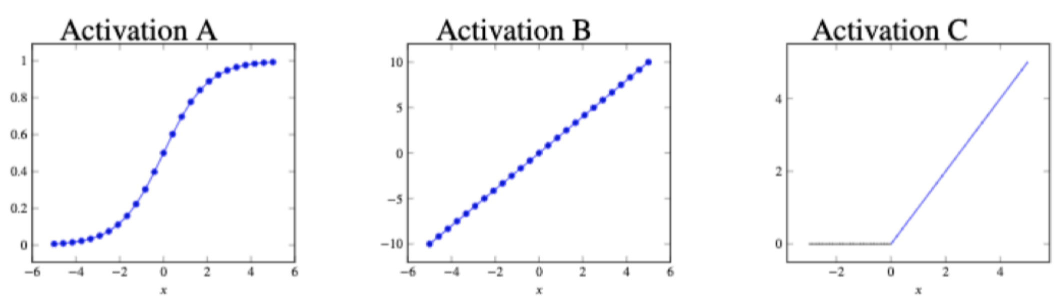
☐ With sufficient amount of data, this neural network can accurately approximate the function $f(x) = \sin(x)$.

☐ A deeper neural network is better at expressing complicated functions than this neural network.

☐ Adding an extra "+" node with bias $b_2$ before (to the left of) the "$*$" node with coefficient $w_1$ expands the set of functions this neural network can represent.

○ None of the above.

6. [**Old exam question**] Assuming that any network configuration is possible, which of the three shown activation functions will allow fitting the functions specified below, with an arbitrary small error $\epsilon$:




(i) $f(x) = \sin x$

☐ A ☐ B ☐ C ○ None

(ii) $f(x) = \begin{cases} 1, & \text{if } x < -1 \\ -x, & \text{if } -1 \leq x < -0 \\ 0, & \text{otherwise} \end{cases}$

☐ A ☐ B ☐ C ○ None

(iii) $f(x) = 1$

☐ A ☐ B ☐ C ○ None