

Exercises: Inference in Bayesian networks

- 1. Given the Bayesian network below, calculate marginal and conditional probabilities $P(\neg x_3)$ and $P(x_2 | \neg x_3)$ by using:
 - (a) the method of inference by enumeration,
 - (b) the method of variable elimination.

$\mathbf{P}(Z)$	(X_1)
x_1	0.4
$\neg x_1$	0.6

$\mathbf{P}($	$\overline{X_3 \mid x}$	$_{2})$
x_2	x_3	0.2
x_2	$\neg x_3$	0.8
$\neg x_2$	x_3	0.3
$\neg x_2$	$\neg x_3$	0.7



$\mathbf{P}(\mathbf{r})$	$X_2 \mid X$	(1)
x_1	x_2	0.8
x_1	$\neg x_2$	0.2
$\neg x_1$	x_2	0.5
$\neg x_1$	$\neg x_2$	0.5

$\mathbf{P}(\mathbf{r})$	$X_4 \mid X$	(2)
x_2	x_4	0.8
x_2	$\neg x_4$	0.2
$\neg x_2$	x_4	0.5
$\neg x_2$	$\neg x_4$	0.5

2. Suppose a burglary alarm which can function in its basic mode and with an additional ultrasonic sensor (which can be turned on or off). The network in Figure 2 represents a complete burglar alarm and notification system, with the following binary random variables: B (burglary happens when B = 1), U (ultrasonic sensor is on when U = 1), A (alarm sounds when A = 1) and N (neighbor calls when N = 1).



When the ultrasonic sensor is active, the probability that alarm is properly activated by burglary is increased from x to y, but the probability of false alarm is also increased, from ε to 2ε . (When the ultrasonic sensor is not active, the probability of false alarm is ε , and with ultrasonic sensor on, the false alarm appears with probability 2ε). Prior probability of burglary in the neighborhood where the house is located is b and the probability that the ultrasonic sensor is active is u.

- (a) Write the joint probability for the network in Fig. 2 (a). Write also a table, which displays conditional probabilities of A given B and U.
- (b) Express the probability that there is a burglary if alarm goes on in terms of x, y, u, ε and b.
- (c) Suppose that the neighbor calls to report the alarm with probability n when the alarm is on and never when the alarm is off. Express the probability that there is a burglary if the neighbor calls and if the ultrasonic sensor is off.
- (d) Extend the network from Figure 2(a) so that it can represent the following statements:
 - (i) the alarm can be activated by pets;
 - (ii) when the alarm goes on, an SMS notification is sent automatically via the Internet.

3. PacLabs has just created a new type of mini power pellet that is small enough for Pacman to carry around with him when he's running around mazes. Unfortunately, these mini-pellets don't guarantee that Pacman will win all his fights with ghosts, and they look just like the regular dots Pacman carried around to snack on.

Pacman just ate a snack (P), which was either a mini-pellet (p), or a regular dot $(\neg p)$, and is about to get into a fight (W), which he can win (w) or lose $(\neg w)$. Both these variables are unknown, but fortunately, Pacman is a master of probability. He knows that his bag of snacks has 5 mini-pellets and 15 regular dots. He also knows that if he ate a mini-pellet, he has a 70% chance of winning, but if he ate a regular dot, he only has a 20% chance.

- (a) What is P(w), the marginal probability that Pacman will win?
- (b) Pacman won! Hooray! What is the conditional probability $P(p \mid w)$ that the food he ate was a mini-pellet, given that he won?
- (c) Pacman can make better probability estimates if he takes more information into account. First, Pacman's breath, B, can be bad (b) or fresh $(\neg b)$. Second, there are two types of ghost (M): mean (m) and nice $(\neg m)$. Pacman has encoded his knowledge about the situation in the following probability distribution

$$\mathbf{P}(M, P, B, W) = \mathbf{P}(M)\mathbf{P}(P)\mathbf{P}(W \mid M, P)\mathbf{P}(B \mid P).$$

Based on the given probability distribution, in the box below draw the corresponding Bayesian network.

	$\mathbf{P}($	(M)]
	m	0.5	1
	$\neg m$	0.5	1
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	$\overline{\mathbf{P}(W)}$	M, P)
m	p	w	0.60
m	p	$\neg w$	0.40
m	$\neg p$	w	0.10
m	$\neg p$	$\neg w$	0.90
$\neg m$	p	w	0.80
$\neg m$	p	$\neg w$	0.20
$\neg m$	$\neg p$	w	0.30

- (d) Just based on the structure, which of the following are guaranteed to be true? Explain your answers.
 - (1) $W \perp B$ (2) $W \perp B \mid P$ (3) $M \perp B$ (4) $M \perp B \mid P$

 $\neg m$

 $\neg p$

 $\neg w \mid 0.70$