ARTIFICIAL INTELLIGENCE (E016350)
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Exercises: Solving problems by searching

1. [R&N, 3.17] Which of the following are true and which are false? Explain your answers.

   (a) Depth-first search always expands at least as many nodes as A^* search with an admissible heuristic.

   (b) $h(n) = 0$ is an admissible heuristic for the 8-puzzle.

   (c) A^* search is of no use in robotics because percepts, states and actions are continuous.

   (d) Breadth-first search is complete even if zero step costs are allowed.

   (e) Assume that a rook can move on a chessboard any number of squares in a straight line, vertically or horizontally, but cannot jump over other pieces. Manhattan distance is an admissible heuristic for the problem of moving the rook from square A to square B in the smallest number of moves.

2. [R&N, 3.33] n vehicles occupy squares $(1, 1)$ through $(n, 1)$ (i.e., the top row) of an $n \times n$ grid. The vehicles must be moved to the bottom row but in reverse order; so the vehicle i that starts in $(i, 1)$ must end up in $(n - i + 1, n)$. On each time step, every one of the n vehicles can move one square up, down, left or right, or stay put; if a vehicle stays put, one other adjacent vehicle (but not more than one) can hop over it. Two vehicles cannot occupy the same square.

   (a) Calculate the size of the state space as a function of n.

   (b) Calculate the branching factor as a function of n.

   (c) Suppose that vehicle i is at (x_i, y_i); write a nontrivial admissible heuristic h_i for the number of moves it will require to get to its goal location $(n - i + 1, n)$, assuming no other vehicles are on the grid.

   (d) Which of the following are admissible heuristics for the original problem and why? (Note: h_i is the heuristic you came up with in the previous part.)

   A. $h = \min\{h_1, \ldots, h_n\}$ B. $h = \max\{h_1, \ldots, h_n\}$ C. $h = \sum_{i=1}^{n} h_i$

3. If $f(s)$, $g(s)$ and $h(s)$ are all admissible heuristics then which of the following are also guaranteed to be admissible heuristics:

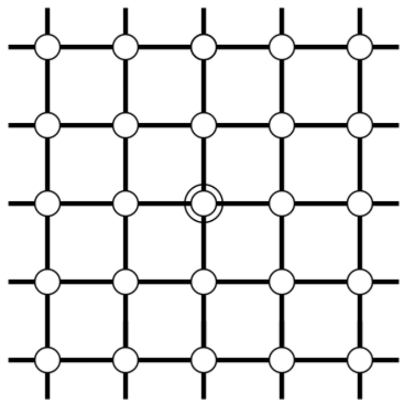
   (a) $f(s) + g(s) + h(s)$

   (b) $f(s)/6 + g(s)/3 + h(s)/2$

   (c) $\min(f(s), g(s), h(s))$

   (d) $\max(f(s), g(s), h(s))$

4. **(Old exam question)** Consider the unbounded version of the regular 2D grid. The start state is at the origin $(0, 0)$, and the goal state is at (x, y).



   (a) What is the branching factor b in this state space?

   (b) How many distinct states are there at depth k (for $k > 0$)?
   A. 4^k B. $4k$ C. $4k^2$

   (c) Breadth-first search *without* repeated-state checking *expands* at most
   A. $(4^{m+n+1} - 1)/3 - 1$ B. $4(m+n) - 1$ C. $2(m+n)(m+n+1) - 1$
   nodes before terminating, where $m = |x|$, $n = |y|$.

   (d) Breadth-first search *with* repeated-state checking *expands* at most
   A. $(4^{m+n+1} - 1)/3 - 1$ B. $4(m+n) - 1$ C. $2(m+n)(m+n+1) - 1$
   nodes before terminating, where $m = |x|$, $n = |y|$.

   (e) Define the heuristic h as a Manhattan distance for a state at (u, v) and write the corresponding expression. Explain!

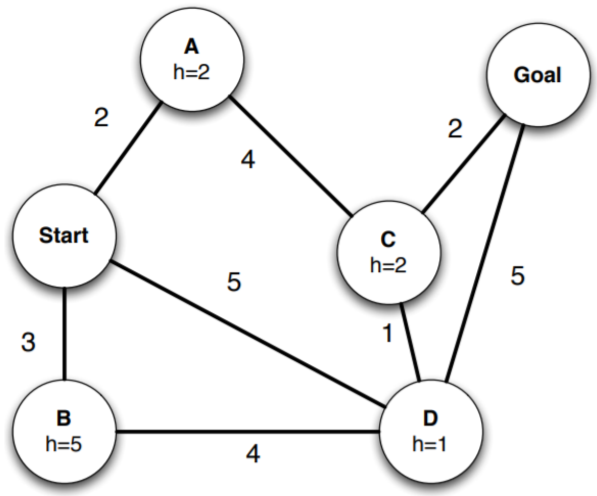
   (f) *True/False*: A^* search with repeated state checking using h expands $\mathcal{O}(x + y)$ nodes before terminating.

   (g) Does h remain admissible if some links are removed? If not, give a nontrivial admissible heuristic for this search problem.

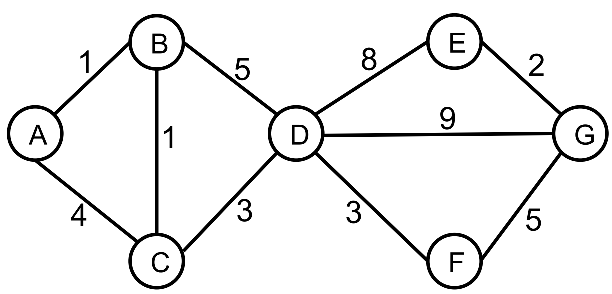
   (h) Does h remain admissible if some links are added between nonadjacent states? If not, give a nontrivial admissible heuristic for this search problem.

5. For each of the following graph search strategies, work out the order in which states are expanded, as well as the path returned by graph search. In all cases, assume ties resolve in such a way that states with earlier alphabetical order are expanded first. The start and goal state are S and G, respectively. Remember that in graph search, a state is expanded only once.

   (a) Depth-first search.
   (b) Breadth-first search.
   (c) Uniform cost search.
   (d) Greedy search with the heuristic h shown on the graph.
   (e) A^* search with the same heuristic h.



6. Consider the state space graph shown below.



Node	h_1	h_2
A	9.5	10
B	9	12
C	8	10
D	7	8
E	1.5	1
F	4	4.5
G	0	0

A is the start state and G is the goal state. The costs for each edge are shown on the graph. Each edge can be traversed in both directions. Note that the heuristic h_1 is consistent but the heuristic h_2 is not consistent.

   (a) For each of the following graph search strategies (do not answer for tree search), mark which, if any, of the listed paths it could return. Note that for some search strategies the specific path returned might depend on tie-breaking behavior. In any such cases, make sure to mark all paths that could be returned under some tie-breaking scheme.

Search Algorithm	A-B-D-G	A-C-D-G	A-B-C-D-F-G
Depth first search			
Breadth first search			
Uniform cost search			
A^* search with heuristic h_1			
A^* search with heuristic h_2			

(b) Suppose you are completing the new heuristic function h_3 shown below. All the values are fixed except $h_3(B)$.

Node	A	B	C	D	E	F	G
h_3	10	?	9	7	1.5	4.5	0

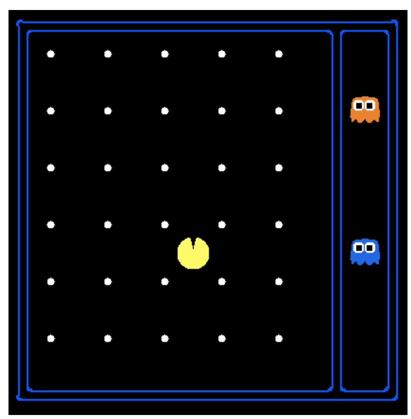
For each of the following conditions, write the set of values that are possible for $h_3(B)$. For example, to denote all non-negative numbers, write $[0, +\infty]$, to denote the empty set, write $\emptyset$, and so on.

1. What values of $h_3(B)$ make h_3 admissible?
2. What values of $h_3(B)$ make h_3 consistent?
3. What values of $h_3(B)$ will cause A^* graph search to expand node A, then node C, then node B, then node D in order?

Remark

As mentioned in the theory lesson not all admissible heuristics are consistent. In practice it is usually assumed that admissible heuristics are consistent, implying that consistency is a desirable attribute. In the AI textbook “Artificial Intelligence: A Modern Approach”, Russell and Norvig write that “although consistency is a stricter requirement than admissibility, one has to work quite hard to concoct heuristics that are admissible but not consistent”. In the following exercise we will present an example of an admissible heuristic that is not consistent. Apart from that we will show that if a heuristic is consistent, then it also has to be admissible.

7. **(State space size)** For the Pacman world shown in the image on the right, assume



- Agent positions: 120
- Food count: 30
- Ghost positions: 12
- Agent facing: NSEW

How many world states are there? How many states for route finding?How many states for “eat-all-dots”?

8. **(Old exam question)** For the 8-queens problem, hill climbing succeeds to find a solution in 14% of cases (and thus it fails in 86% of the cases). When it succeeds, it takes 4 steps on average and when it fails it takes 3 steps on average. How many steps will take on average to solve the problem with a random-restart hill climbing algorithm?

9. **(Optional theoretical exercise)** [Adapted from R&N, 3.37]

   (a) Prove that if a heuristic is consistent, it must be admissible.

   (b) Construct an admissible heuristic that is not consistent.

   (c) We know that A^* tree search is optimal if the heuristic h is admissible, while the A^* graph search is optimal if the heuristic h is consistent. Give a simple example that shows that A^* graph search with inconsistent but admissible heuristic h is not necessarily the optimal search strategy.