ARTIFICIAL INTELLIGENCE (E016350)
GHENT UNIVERSITY
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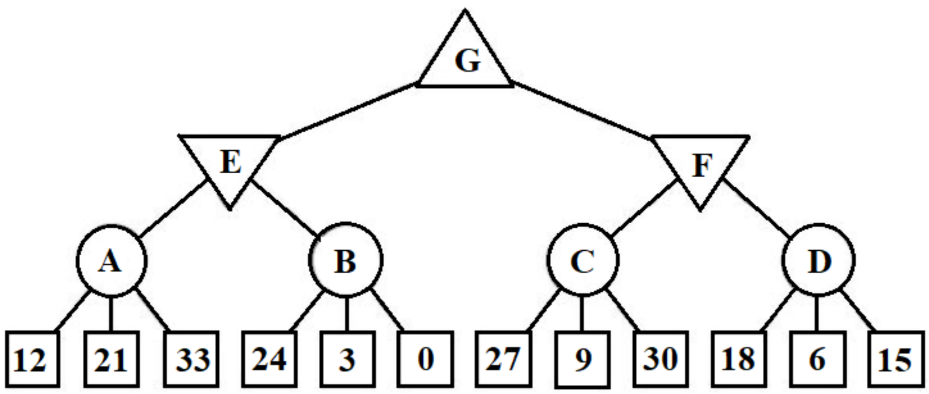
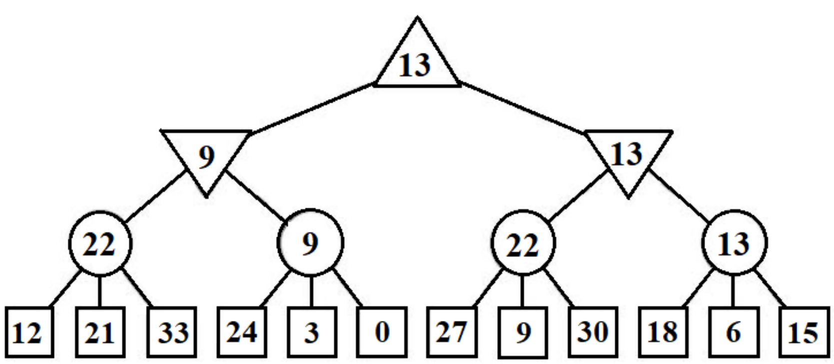
Solutions: Game Playing

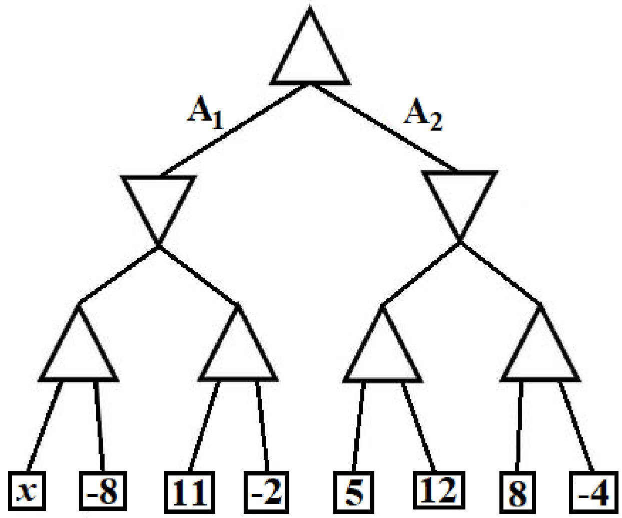
1. On the game-tree shown below, the MAX player is presented with triangles facing up, the MIN player is presented with a triangles facing down, while the CHANCE player is presented with circles. Assuming that players MAX i MIN play optimally, and player CHANCE moves randomly with the same probability, determine the values in nodes A, B, C, D, E, F and G.



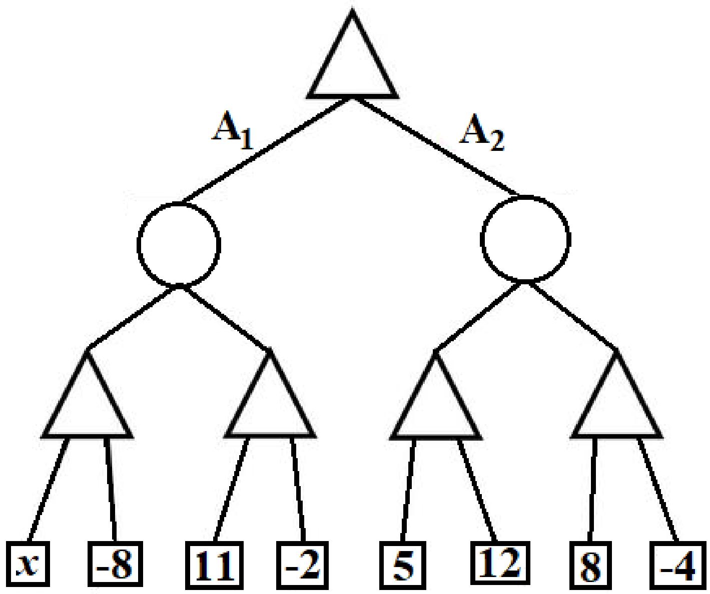
Solution:
By using the simple rules of Expectiminimax algorithm we can easily obtain the following game-tree below.



2. (a) Assuming that MAX player is presented with triangles facing up, and the MIN player presented with triangles facing down, for which value x would the player MAX select the action A_1?



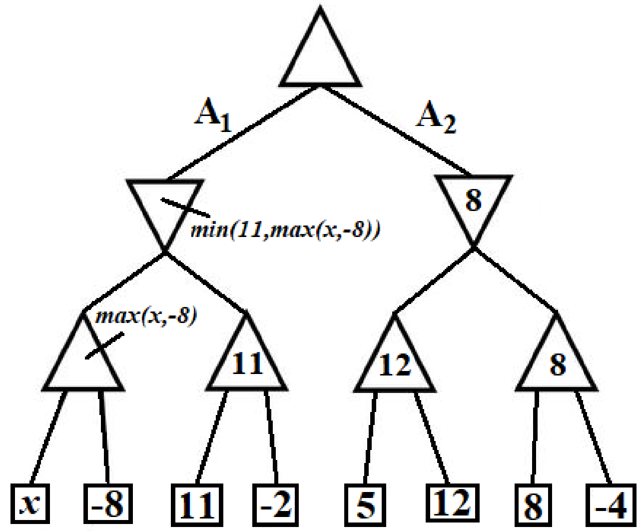
(b) Assuming that the MAX player is presented with triangles facing up, and the player CHANCE (who moves randomly with the same probability) presented with circles, for which value x would the player MAX select the action A_1?



Solution:
First, we fill in the values that don't depend on the node with x value, both in the (a) and (b) example. Afterwards we discuss the conditions under which is guaranteed that the action A_1 will be selected.

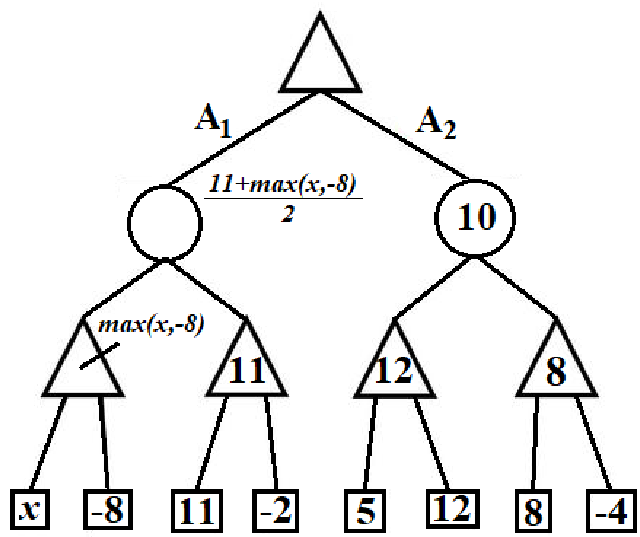
(a) In the first example the action A_1 will be taken if:

$$\min(11, \max(x, -8)) > 8 \implies x \in (8, +\infty).$$



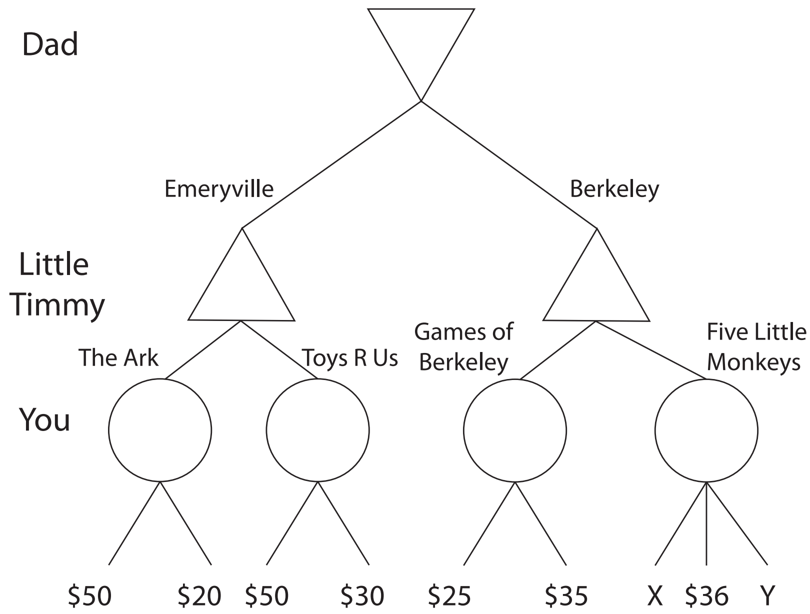
(b) In the second example we obtain similarly that the action A_1 will be taken if:

$$\frac{11 + \max(x, -8)}{2} > 10 \iff x \in (9, +\infty).$$



3. Your little brother Timmy has a birthday and he was promised a toy. However, Timmy has been misbehaving lately and Dad thinks he deserves the least expensive present. Timmy, of course, wants the most expensive toy. Dad will pick the city from which to buy the toy, Timmy will pick the store and you get to pick the toy itself. You don't want to take sides so you decide

to pick a toy at random. All prices (including X and Y) are assumed to be nonnegative.



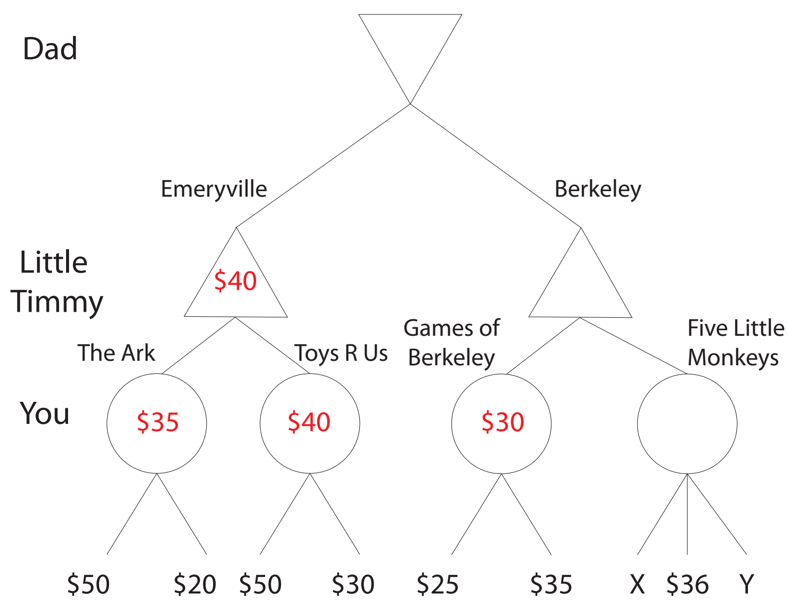
(a) Fill in the values of all the nodes that don't depend on X or Y.

(b) What values of X will make Dad pick Emeryville regardless of the price of Y?

(c) We know that Y is at most \$30. What values of X will result in a toy from Games of Berkeley regardless of the exact price of Y?

Solution:

(a)



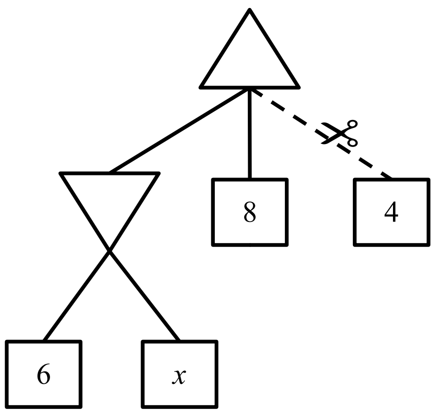
(b) Dad will pick Emeryville if the value of the Berkeley node is more than the value of the Emeryville node (\$40), that is if:

$$\frac{x + y + 36}{3} > 40 \iff x > 84 - y \iff x > 84.$$

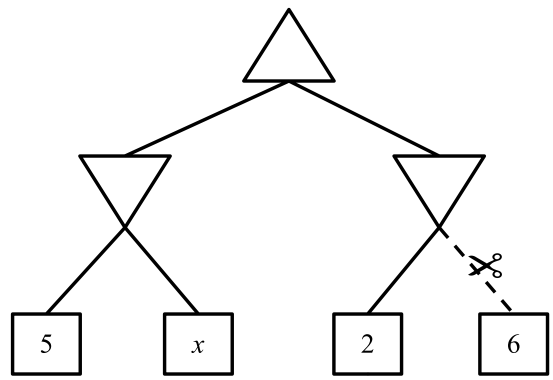
(c) Games of Berkeley will be chosen if the value of the "Five Little Monkeys" node is less than the value of the "Games of Berkeley" node ($30):

$$\frac{x+y+36}{3} < 30 \quad \Longleftrightarrow \quad x < 54 - y \quad \Longleftrightarrow \quad x < 24.$$

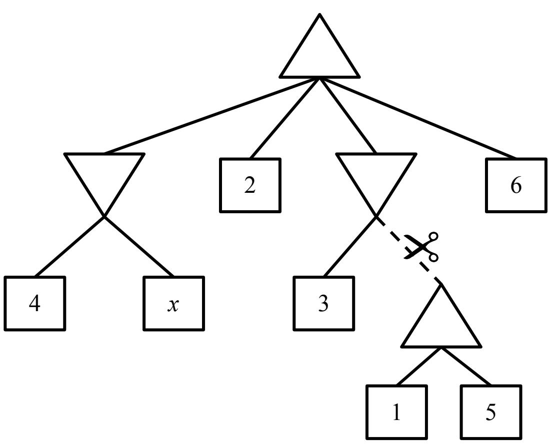
4. For each of the game-trees shown below, state for which values of x the dashed branch with the scissors will be pruned. If the pruning will not happen for any value of x write "none". If pruning will happen for all values of x write "all".



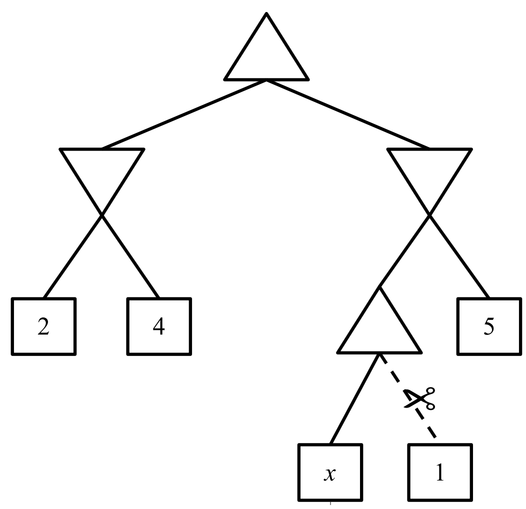
(a) Tree 1. Answer: ______________



(b) Tree 2. Answer: ______________



(c) Tree 3. Answer: ______________



(d) Tree 4. Answer: ______________

Solution:
We are assuming that nodes are evaluated left to right and ties are broken in favor of the latter nodes. A different evaluation order would lead to different interval bounds, while a different tie breaking strategies could lead to strict inequalities ($>$ instead of $\geq$).

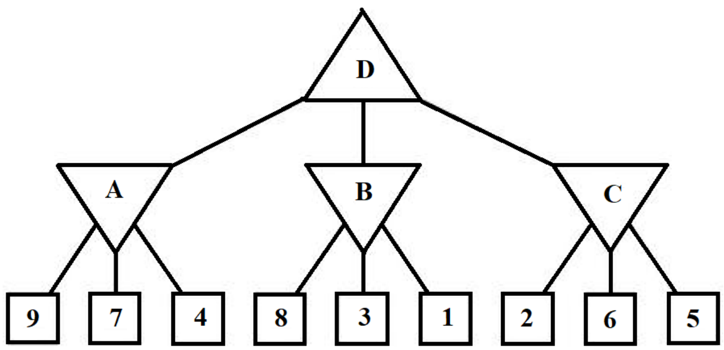
(a) $None$ (b) $x \geq 2$

(c) $x \geq 3$ (d) $None$

5. Perform the minimax algorithm on the figure below:

   (a) Without $\alpha - \beta$ pruning.

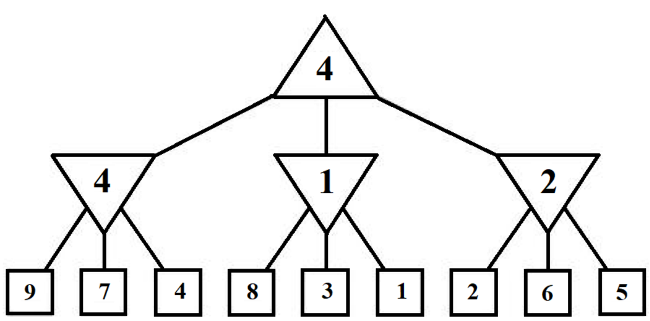
   (b) With $\alpha - \beta$ pruning.



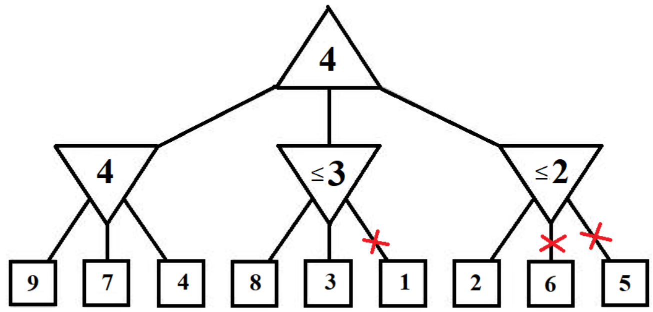
Additionally: Check your answers and generate many other examples on: *Alpha-Beta Pruning Practice*.

Solution:

(a)



(b)



6. Prove the following assertion: For every game tree, the utility obtained by MAX using minimax decisions against a suboptimal MIN will be never be lower than the utility obtained playing against an optimal MIN. Can you come up with a game tree in which MAX can do still better using a suboptimal strategy against a suboptimal MIN?

Solution:

Hint: Think how the utilities for MAX will change if MIN is suboptimal and how these will propagate up the tree. Can the resulting utility be smaller? For the question at the end, think of a case where your oponent is suboptimal and in a predictable way. Could you set a trap? How would such a game tree look like? (This was all analysed in detail during the lecture)