

## Probabilistic reasoning over time

1. Consider the following simple Hidden Markov Model with state variables  $X_t$  and observation (evidence) variables  $O_t$ , which are shaded below.



$X_1$	$P(X_1)$
0	0.3
1	0.7
	,

$\zeta_t$	$X_{t+1}$	$\mathbf{P}(X_{t+1} X_t)$
)	0	0.4
)	1	0.6
L	0	0.8
I	1	0.2

$X_t$	$O_t$	$P(O_t X_t)$
0	A	0.9
0	B	0.1
1	A	0.5
1	B	0.5

Suppose that  $O_1 = A$  and  $O_2 = B$  is observed.

(a) Use the Forward algorithm to compute the probability distribution

$$\mathbf{P}(X_2 \mid O_1 = A, O_2 = B).$$

Show your work.

- (b) Use the Viterbi algorithm to compute the maximum probability sequence  $X_1$ ,  $X_2$ . Show your work.
- 2. (Old exam question) Suppose you returned from holidays and due to the current Covid-19 regulations have to stay in quarantine for two weeks. You are staying in a good ventilated, but windowless basement room in your home. To make time go by faster, you want to develop a simple weather forecast system that only gives prognosis for average daily weather (W) characterized as being either sunny (s), rainy (r) or foggy (f), i.e.,  $w \in \{s, rf\}$ .

You will make the system more sophisticated by making use of some indirect evidence that you get by observing the caretaker that visits you each morning. In particular you pay attention to whether the caretaker caries an umbrella or not. You assume that the probability that the caretaker carries an umbrella is 0.1 if the weather is sunny, 0.8 if the weather is rainy and 0.3 if it is foggy.

Based on some statistical data that you could find for Ghent region in Belgium, you set probabilities of tomorrow's weather based on today's weather as in the table below.

		Tomorrow's weather			
		Sunny	Rainy	Foggy	
Today's weather	Sunny	0.8	0.05	0.15	
	Rainy	0.2	0.6	0.2	
	Foggy	0.2	0.3	0.5	

- (a) Draw the corresponding state transition diagram that represents the transition model with the state transition probabilities.
- (b) Specify the sensor model with an appropriate table.
- (c) Suppose first you discard the information that you can obtain by observing the caretaker. What is the probability that it will be rainy two days from now given that today is foggy?
- (d) You have no idea anymore about what the weather was before you landed in the basement and it is your second day there. What is the probability that on this day 2 of your quarantine the weather outside is sunny provided that the caretaker didn't carry the umbrella on day 1 and caries it on day 2?
- (e) On the first 3 days your umbrella observations are: {no\_umbrella, umbrella, umbrella, umbrella}. Find the most probable weather-sequence using the Viterbi algorithm.
- 3. (Old exam question) Sam has bought an AI agent to be his companion helping him to keep good mood during the exam period. The AI agent can analyse facial expressions and adjust accordingly lighting in the room, music and tell some jokes to stimulate this way different activities like studying or relaxing. During the first day, the agent is just observing Sam's facial expressions and based on average observations in given time intervals infers the type of Sam's activity in those corresponding intervals.

Suppose that the time interval is set to one hour and that Sam's activity during each time interval is either *Studying* (S) or playing *Video games* (V). His facial expression that the agent observes is either *Grinning* (G) or *Frowning* (F).

(a) Based on the available information from the given state space diagram, write the tables for the transition and sensor models and any other if available from the problem description. Use  $D_i$  to denote the random variable (RV) representing what Sam is doing in hour *i* and denote by  $O_i$  the observation RV in that time slot.



(b) If in the second hour Sam is *Studying*, what's the probability that in the fourth hour he is playing *Video games*? Assume the model from (a).

- (c) Under the model in (a), what is the probability that Sam is studying in the second hour if the observation sequence for the first two hours is {*Grinning*, *Frowning*}?
- (d) Using the same model, the agent recalculates the probabilities of Sam's activity during hour 2, after observing that his facial expression during hour 3 is *Grinning*. Show this calculation and the updated probabilities of the activities in the specified time slot. Comment on this result.
- (e) Suppose now that Sam's activities also include ChattingWithFriend (C) and the set of his observable facial expressions is extended with two others: Excited (E) and Bored (B). The transition model and the sensor model are given by the two matrices below:

:	Studying V	ideoGam	es Chatting	3	Studying S	VideoGames V	Chatting C
	s F o 1	۷ م	د م	Frowning F	0.05	0.7	0.25
Studying S	0.1	0.2	0.2	Grinning G	0.05	0.15	0.4
VideoGames V	0.1	0.7	0.7	Bored B	0.25	0.05	0.05
Chatting C	0.8	0.1	0.1	Excited E	0.65	0.1	0.3

For example, the probability that Sam is chatting with a friend in the hour i+1 if he was studying during the hour i is  $P(C|S) = T_{3,1} = 0.8$  and the probability that he is playing video games if he was studying the hour before is  $P(V|S) = T_{2,1} = 0.1$ . He is grinning with probability 0.05 while studying and with probability 0.4 while chatting.

The agent observes the following sequence of facial expressions: {*Grinning*, *Excited*, *Frowning*}. Which sequence of activities does it infer by applying the Viterbi algorithm? Write the resulting sequence and mark it on the trellis diagram!