ARTIFICIAL INTELLIGENCE (E016330)
GHENT UNIVERSITY
AY 2024/2025
Prof. Aleksandra Pizurica
Assistant: N. Vercheval
E-mail: ai@lists.ugent.be

Exercises: Probabilistic reasoning - Basics

1. Show from the main principles that $P(a|b \wedge a) = 1$.

2. Consider the set of all possible five-card poker hands dealt fairly from a standard deck of fifty-two cards.

   (a) How many atomic events are there in the joint probability distribution (i.e., how many five-card hands are there)?

   (b) What is the probability of each atomic event?

   (c) What is the probability of being dealt a royal straight flush? Four of a kind?

   Note: A straight flush is a hand that contains five cards of sequential rank, all of the same suit, such as Q♣ J♣ 10 ♣ 9♣ 8♣ (a "queen-high straight flush"). An ace-high straight flush, such as A♠ K♠ Q♠ J♠ 10♠, is called a royal flush or royal straight flush. Four of a kind, also known as quads, is a hand that contains four cards of one rank and one card of another rank, e.g., 5♢ 5♠ 5♡ 5♣ Q♡ ("four of a kind, fives").

3. It is quite often useful to consider the effect of some specific propositions in the context of some general background evidence that remains fixed, rather than in the complete absence of information. The following questions ask you to prove more general versions of the product rule and Bayes' rule, with respect to some background evidence $\mathbf{e}$:

   (a) Prove the conditionalized version of the general product rule:

   $$\mathbf{P}(X, Y \mid \mathbf{e}) = \mathbf{P}(X \mid Y, \mathbf{e})\,\mathbf{P}(Y \mid \mathbf{e}).$$

   (b) Prove the conditionalized version of Bayes' rule on evidence $\mathbf{e}$:

   $$\mathbf{P}(Y \mid X, \mathbf{e}) = \frac{\mathbf{P}(X \mid Y, \mathbf{e})\,\mathbf{P}(Y \mid \mathbf{e})}{\mathbf{P}(X \mid \mathbf{e})}.$$

4. Let us consider the example from the theory class: a domain consisting of just the three Boolean variables *Toothache, Cavity,* and *Catch* (the dentist's nasty steel probe catches in my tooth). The full joint distribution is a $2 \times 2 \times 2$ table as shown below.

	toothache		*¬tootache*	
	catch	*¬catch*	*catch*	*¬catch*
cavity	0.108	0.012	0.072	0.008
¬cavity	0.016	0.064	0.144	0.576

Calculate the following:

(a) $\mathrm{P}(toothache)$;

(b) $\mathbf{P}(Cavity)$;

(c) $\mathbf{P}(Toothache \mid cavity)$;

(d) $\mathbf{P}(Cavity \mid toothache \vee catch)$.

5. Suppose you are given a coin that lands heads with probability x and tails with probability $1 - x$. Are the outcomes of successive flips of the coin independent of each other given that you know the value of x? Are the outcomes of successive flips of the coin independent of each other if you do not know the value of x? Justify your answer.

6. **(Old exam question)** Consider two medical tests, A and B, for a virus. Test A is 95% effective at recognizing the virus when it is present, but has a 10% false positive rate (indicating that the virus is present, when it is not). Test B is 90% effective at recognizing the virus, but has a 5% false positive rate. The two tests use independent methods of identifying the virus. The virus is carried by 1% of all people. Say that a person is tested for the virus using only one of the tests, and that test comes back positive for carrying the virus. Which test returning positive is more indicative of someone really carrying the virus? Justify your answer mathematically.

7. After your yearly checkup, the doctor has bad news and good news. The bad news is that you tested positive for a serious disease and that the test is 99% accurate (i.e., the probability of testing positive when you do have the disease is 0.99, as is the probability of testing negative when you don't have the disease). The good news is that this is a rare disease, striking only 1 in $10,000$ people of your age. Why is it good news that the disease is rare? What are the chances that you actually have the disease?