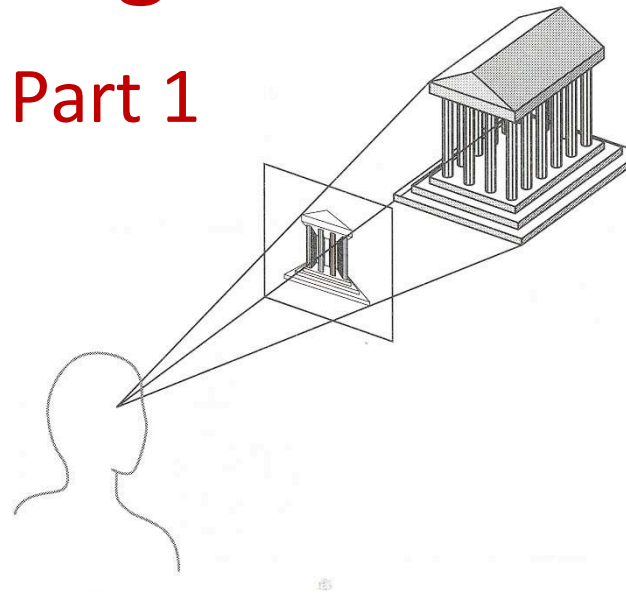


# E016712: Computer Graphics

## Viewing in 3D

### Part 1



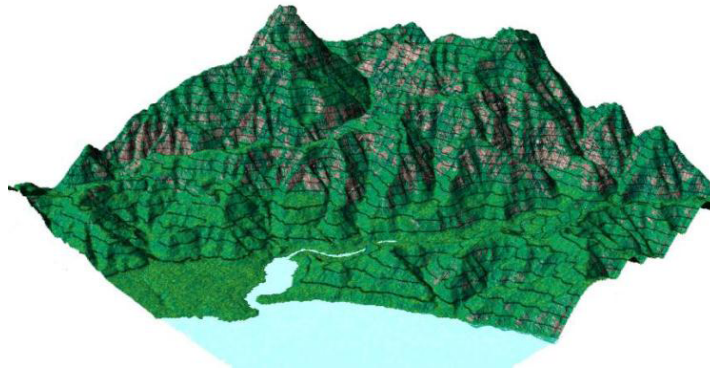
Lecturers: Aleksandra Pizurica and Danilo Babin

# Overview

- Conceptual modelling of the 3D viewing process
- Planar Geometric Projections
- Synthetic Camera Model
- Specifying Arbitrary View in 3D: Examples

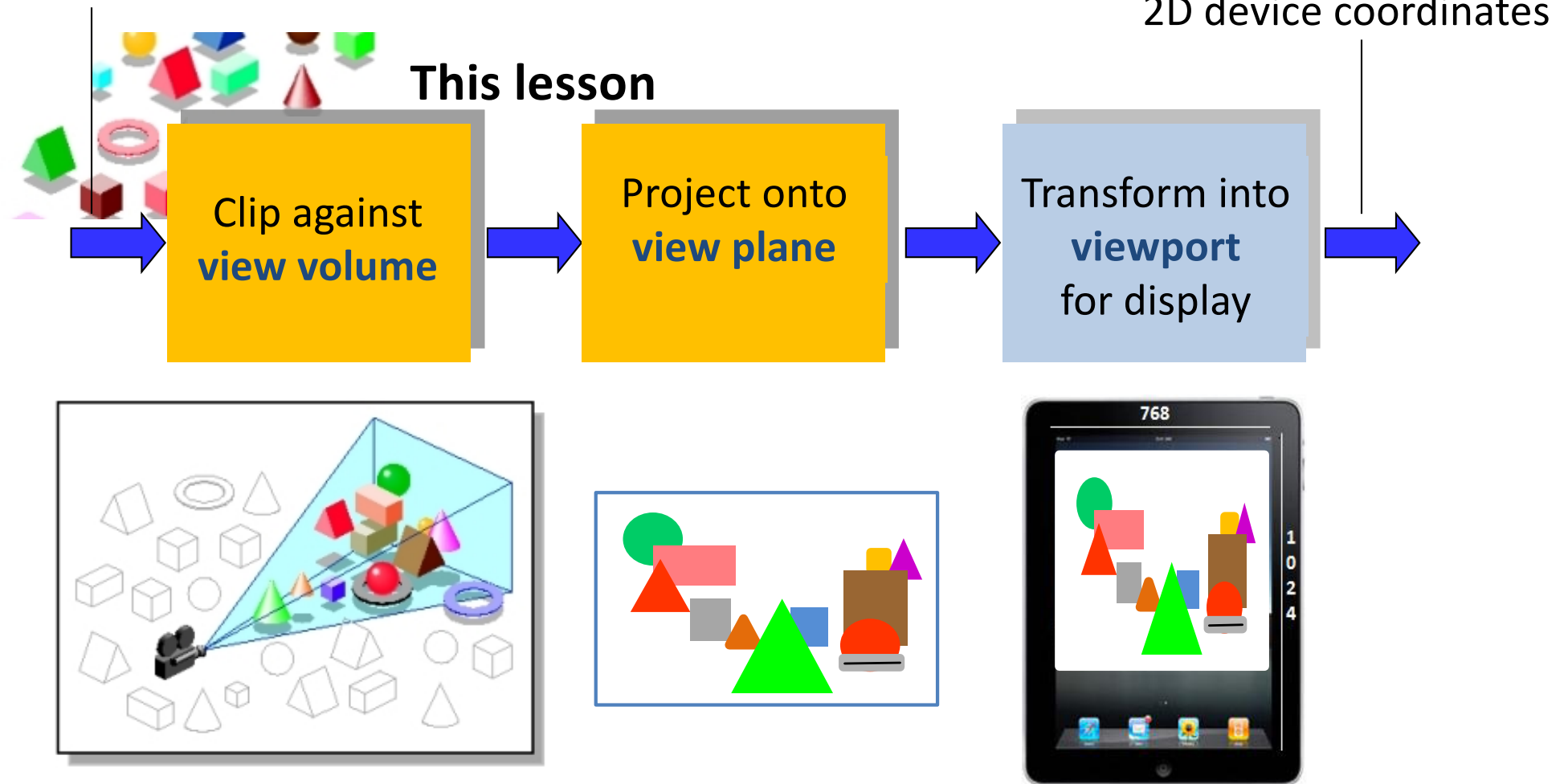
# Viewing in 3D: context

- Create views of 3D objects on 2D display devices



# Conceptual model of the 3D viewing process

3D world coordinates



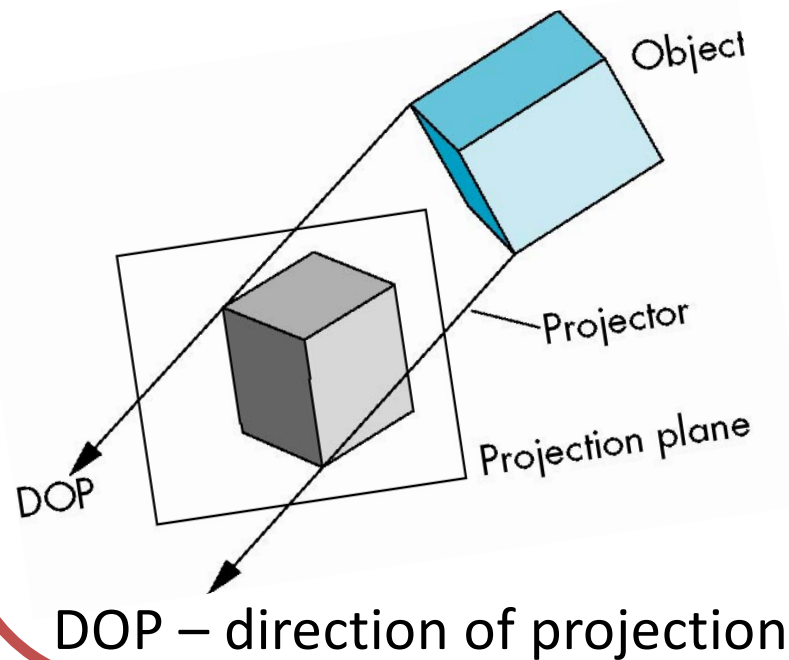
# Planar Geometric Projections



# Planar Geometric Projections

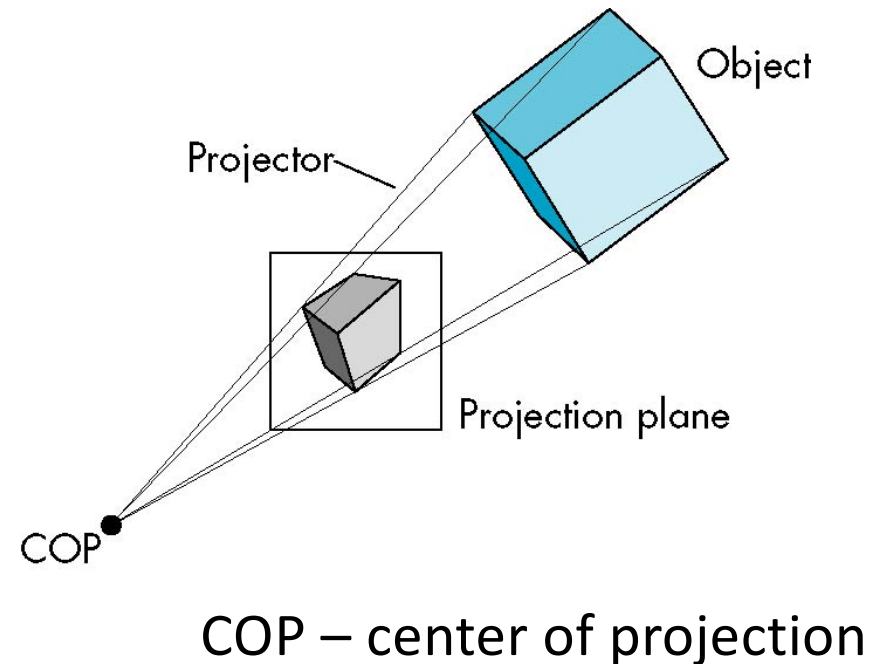
## Parallel projection

Mainly for technical drawings  
(accurate measures)



## Perspective projection

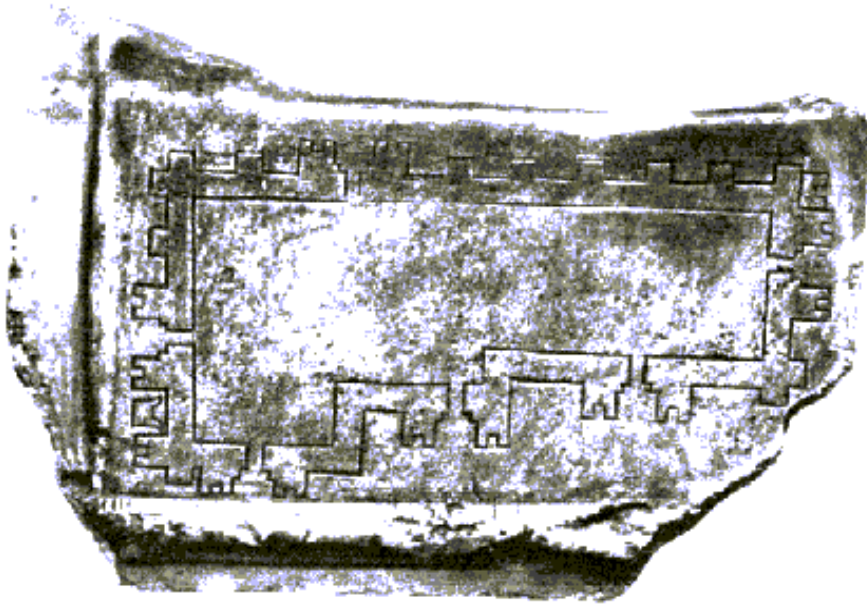
More natural, not convenient  
for accurate measurements



Projection plane = view plane = picture plane

COP = Projection Reference Point (PRP) = eye = view point

# Some historical notes



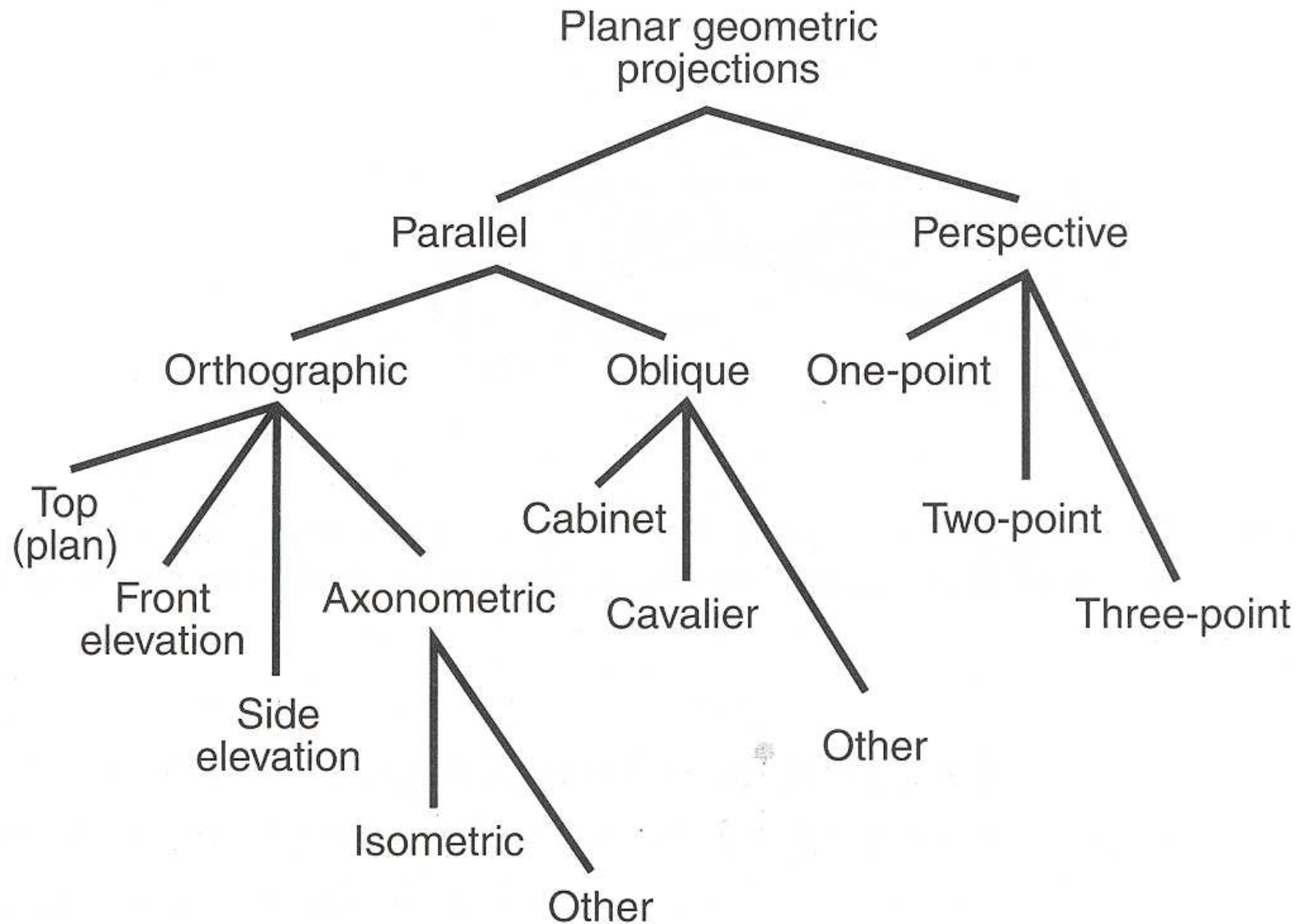
Orthographic projection from  
Mesopotamia, 2150 BC

Ancient theories of vision  
Emission theory: Euclid, Ptolemy  
Intromission theory: Aristotle



Alhazen's book of optics  
Alhazen (Ibn al-Haytham; 965-1040)

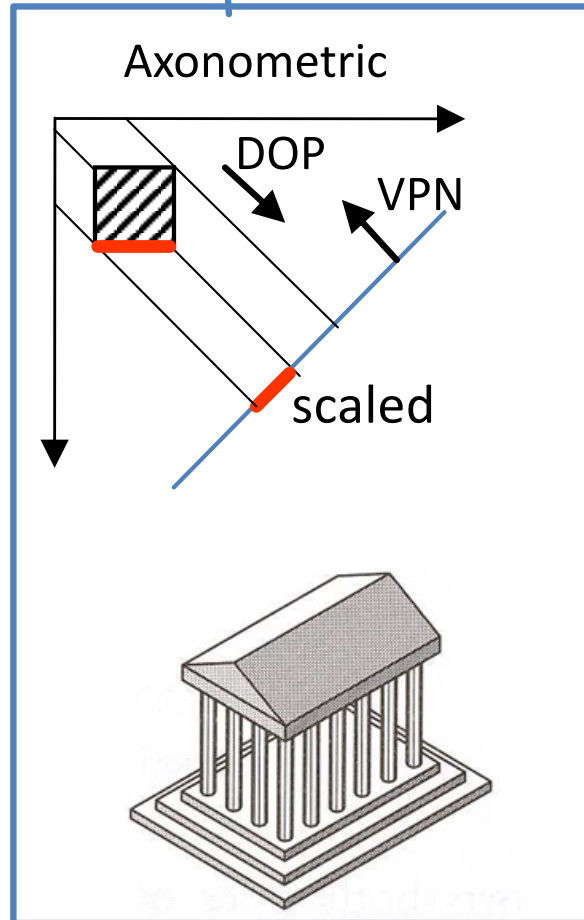
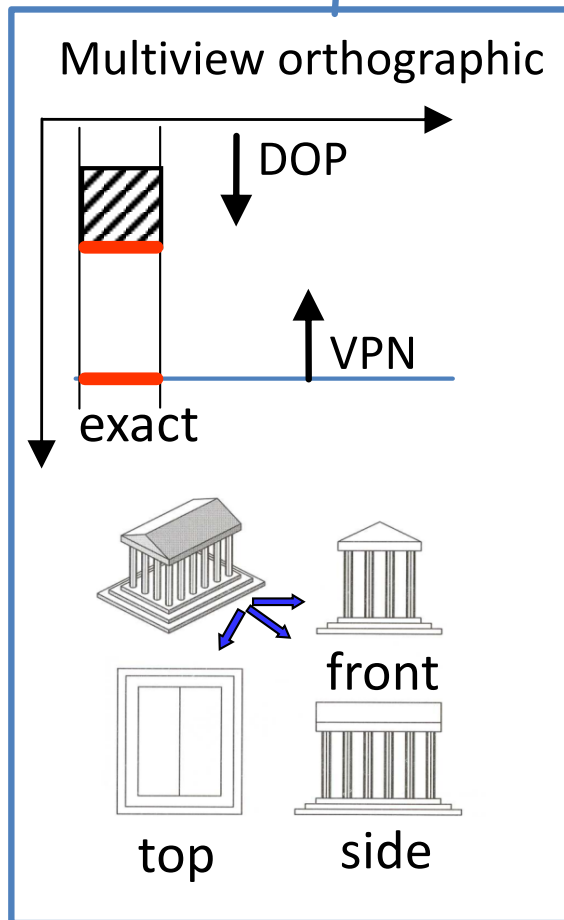
# Projections



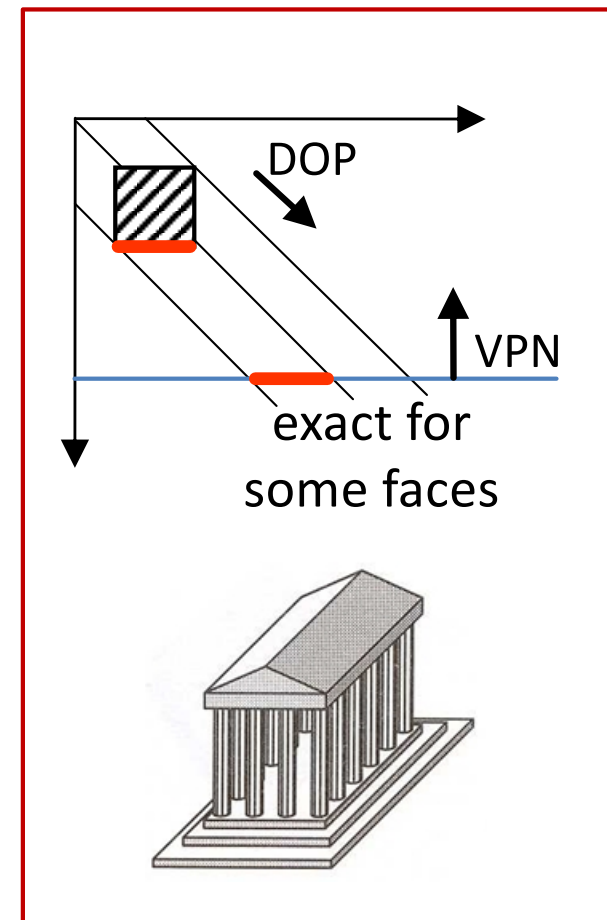


# Parallel projections

## orthographic



## oblique



VPN – view plane normal

DOP – direction of projection

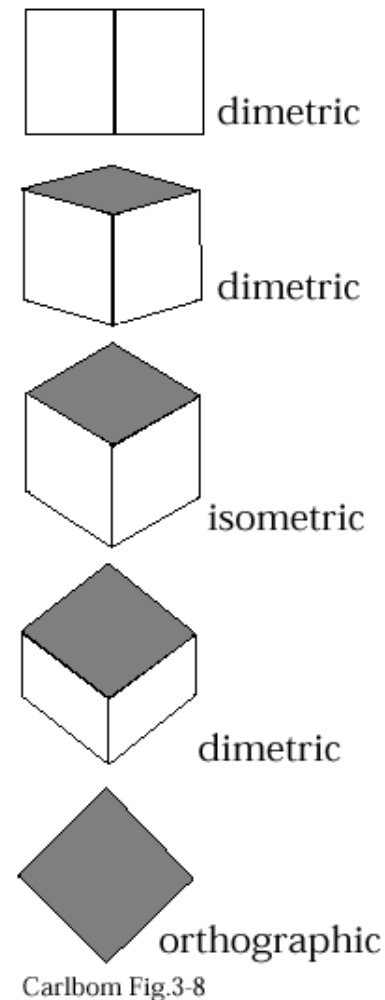
# Parallel projections: Axonometric projection (1)

Three types of axonometric projections:

- **Isometric:** Angles between VPN and any of the three principal axes are equal ( $120^\circ$ ). The same scale ratio applies along each axis
- **Dimetric:** Angles between VPN and two of the principal axes equal; two scale ratios
- **Trimetric:** Different angles between VPN and any of the three principal axes; three scale ratios

Contrasting to multiview orthographic projection, some object faces can be foreshortened

**Foreshortening:** the size of an object's dimensions along the line of sight are relatively shorter than dimensions across the line of sight



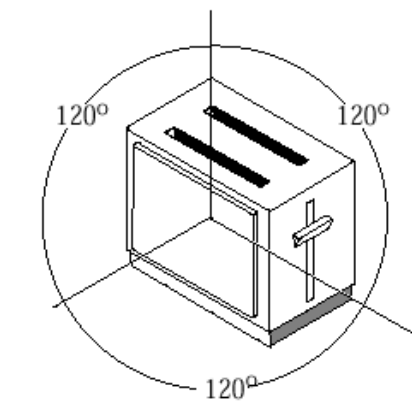
# Parallel projections: Axonometric projection (2)

## Isometric projection

- Often used for catalogue illustrations
- No foreshortening
  - Good for measurements
  - Can appear less realistic
- Used a lot in older video games (1980's) and still now in cases where it is of interest to “see” objects in distance as well as those close up (e.g. strategy, simulation games)



Screen shot of the simulation game LinCity-NG



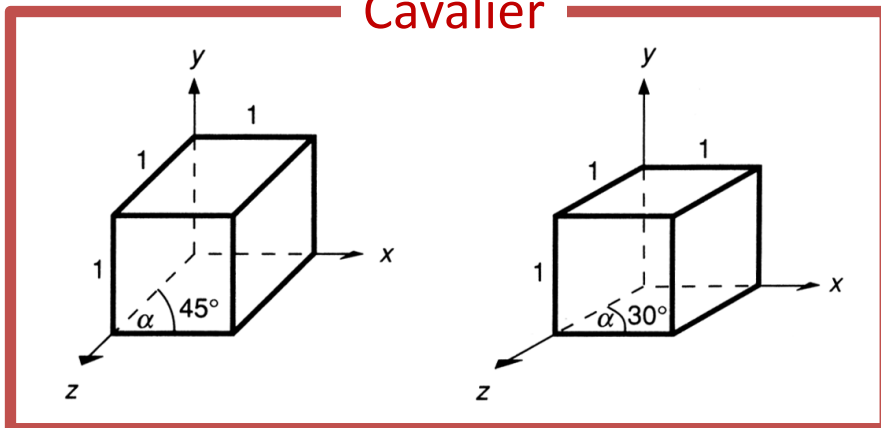
Example

Carlbon Fig.2.2

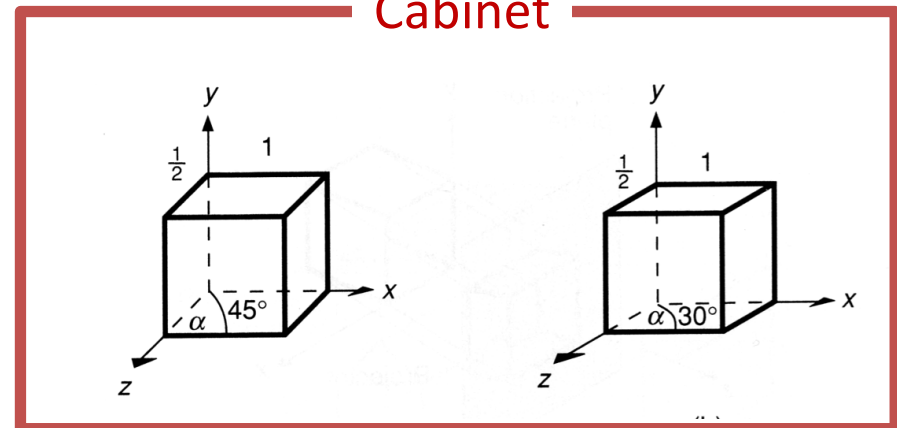
# Parallel projections: Oblique projection (1)

- Two common types: cavalier and cabinet
- **Cavalier**: the direction of the projection makes a 45 degree angle with the projection plane.
  - No foreshortening: the projection of a line perpendicular to the projection plane has the same lengths as the line itself
- **Cabinet**: The direction of the projection makes a 63.4 degree angle with the projection plane.
  - Foreshortening of the z axis, and provides a more “realistic” view

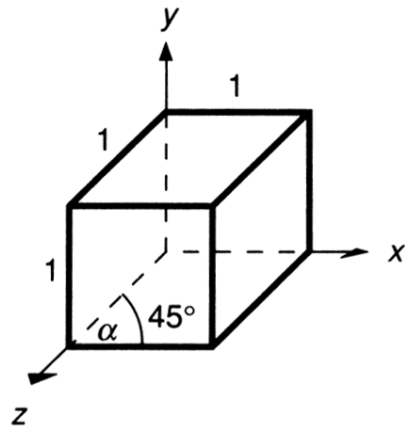
Cavalier



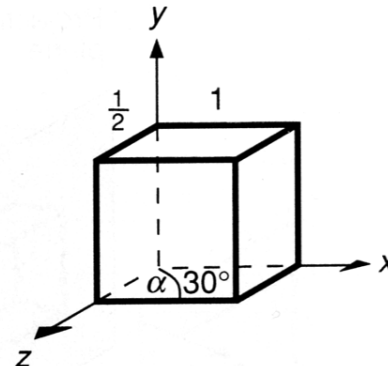
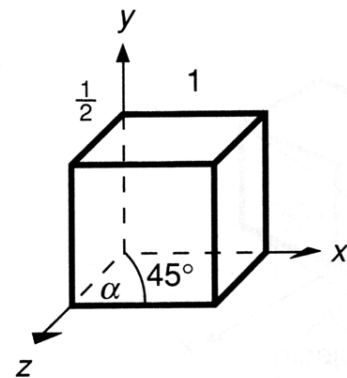
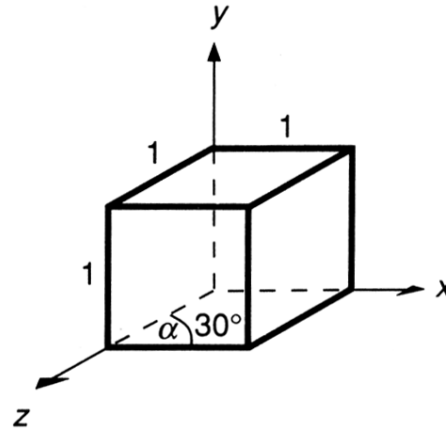
Cabinet



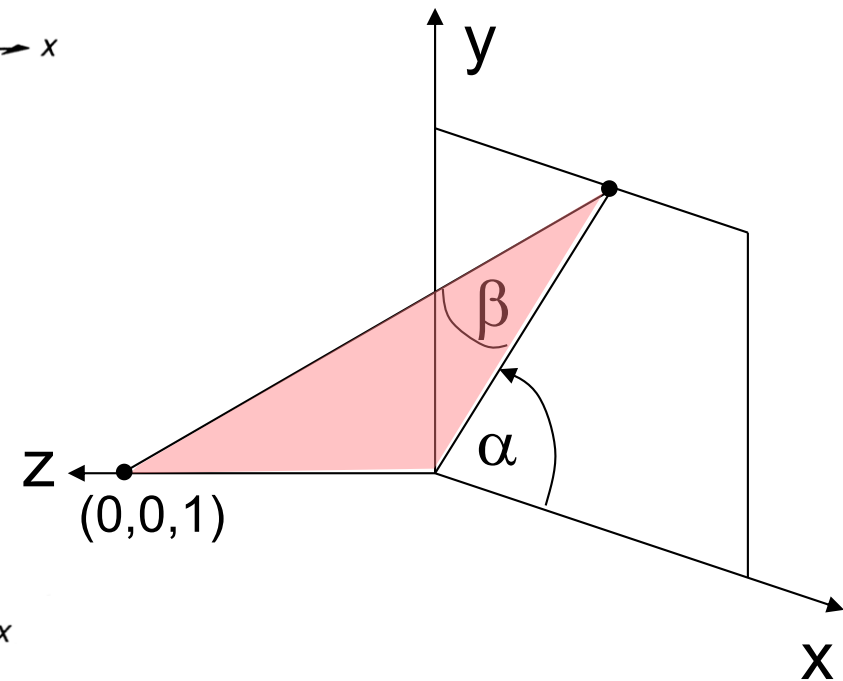
# Parallel projections: Oblique projection (2)



Cavalier:  $\beta = 45^\circ$

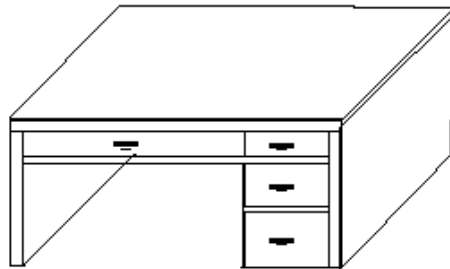


Cabinet:  $\beta = 63.43^\circ$

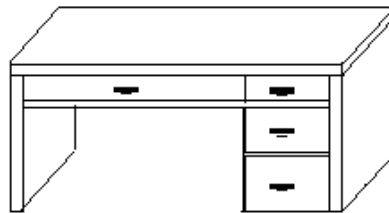


# Parallel projections: Oblique projection (2)

- Used extensively in catalogue illustrations



Cavalier



Cabinet

Carlson Fig. 3-2



Cabinet

- Also used a lot in video games

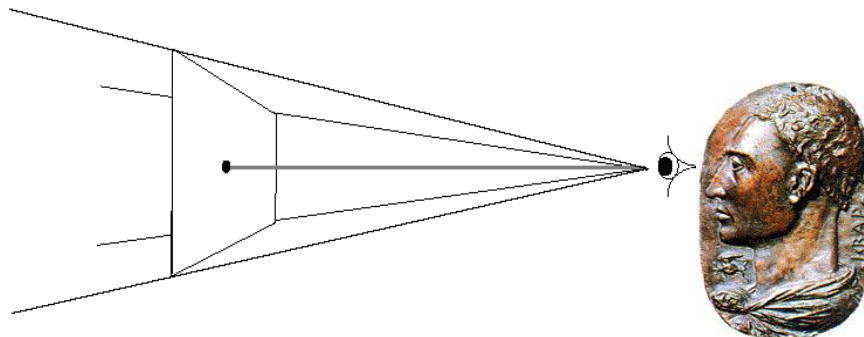
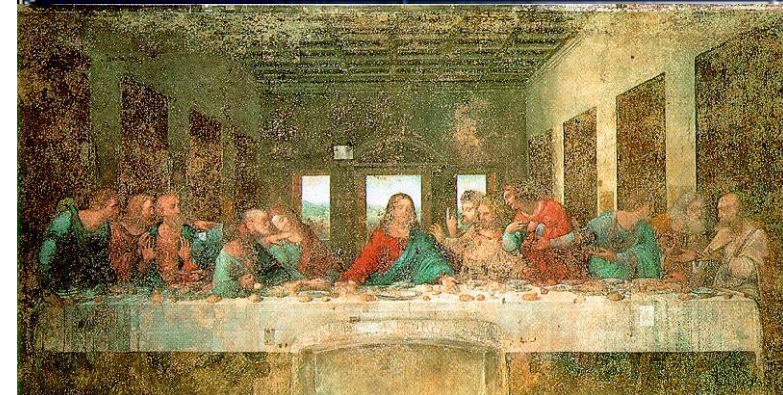
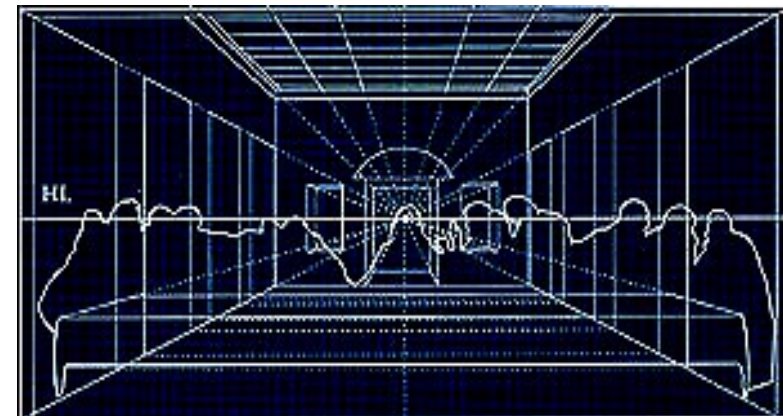
Examples from SimCity – a city building simulation game





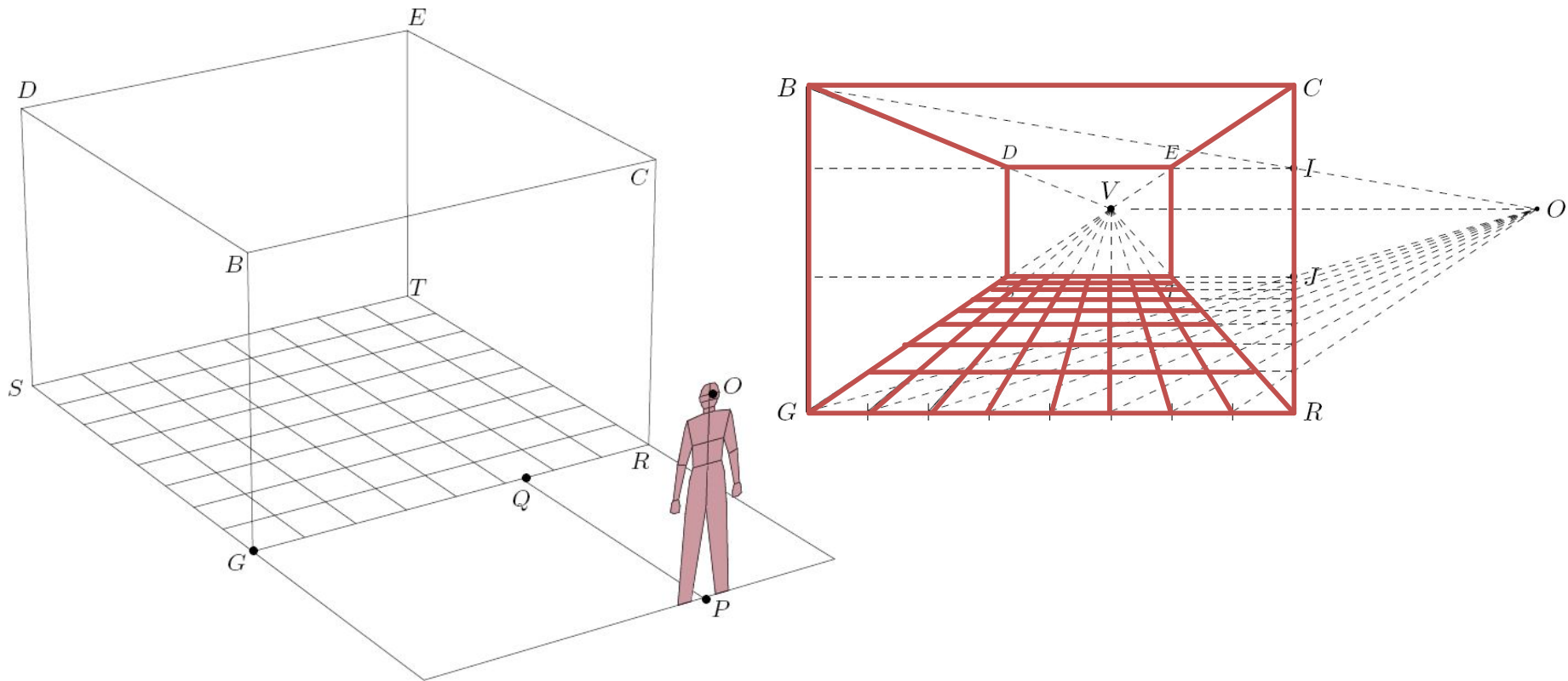
# Perspective projections

- Parallel lines converge to a vanishing point
- Objects that are farther away appear smaller than those that are close → non-uniform foreshortening
- Looks natural, used in fine arts
  - Art-historical context: Alberti, *Della Pittura* (1435), renaissance painters



# Perspective: Art-historical context

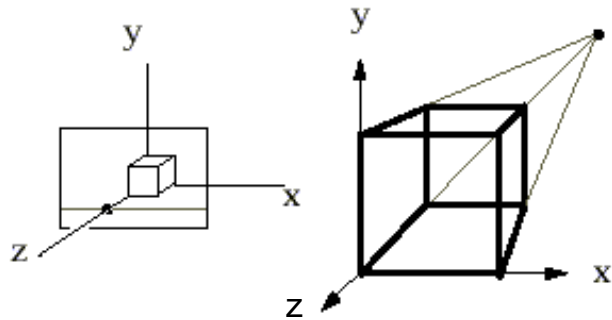
- **Alberti, *Della Pittura* (1435):**
  - First published technical work on perspective.
  - Gives a “recipe” to artists how to create a perspective view of a 3D scene



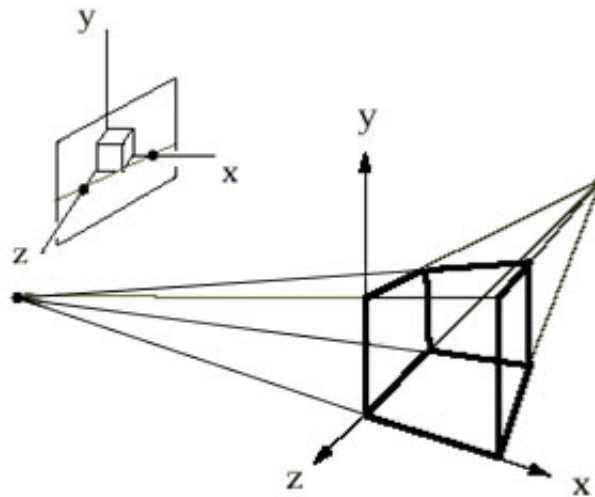


# Categorization of perspective projections

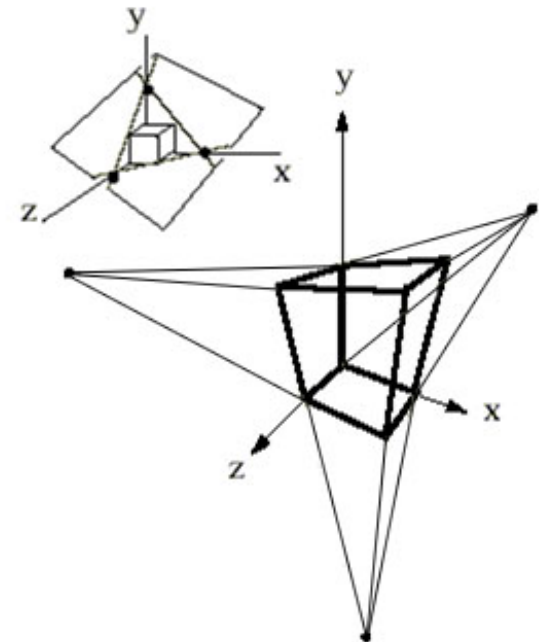
Parallel lines not parallel to view plane converge to a vanishing point:



**One Point Perspective**  
(z-axis vanishing point)



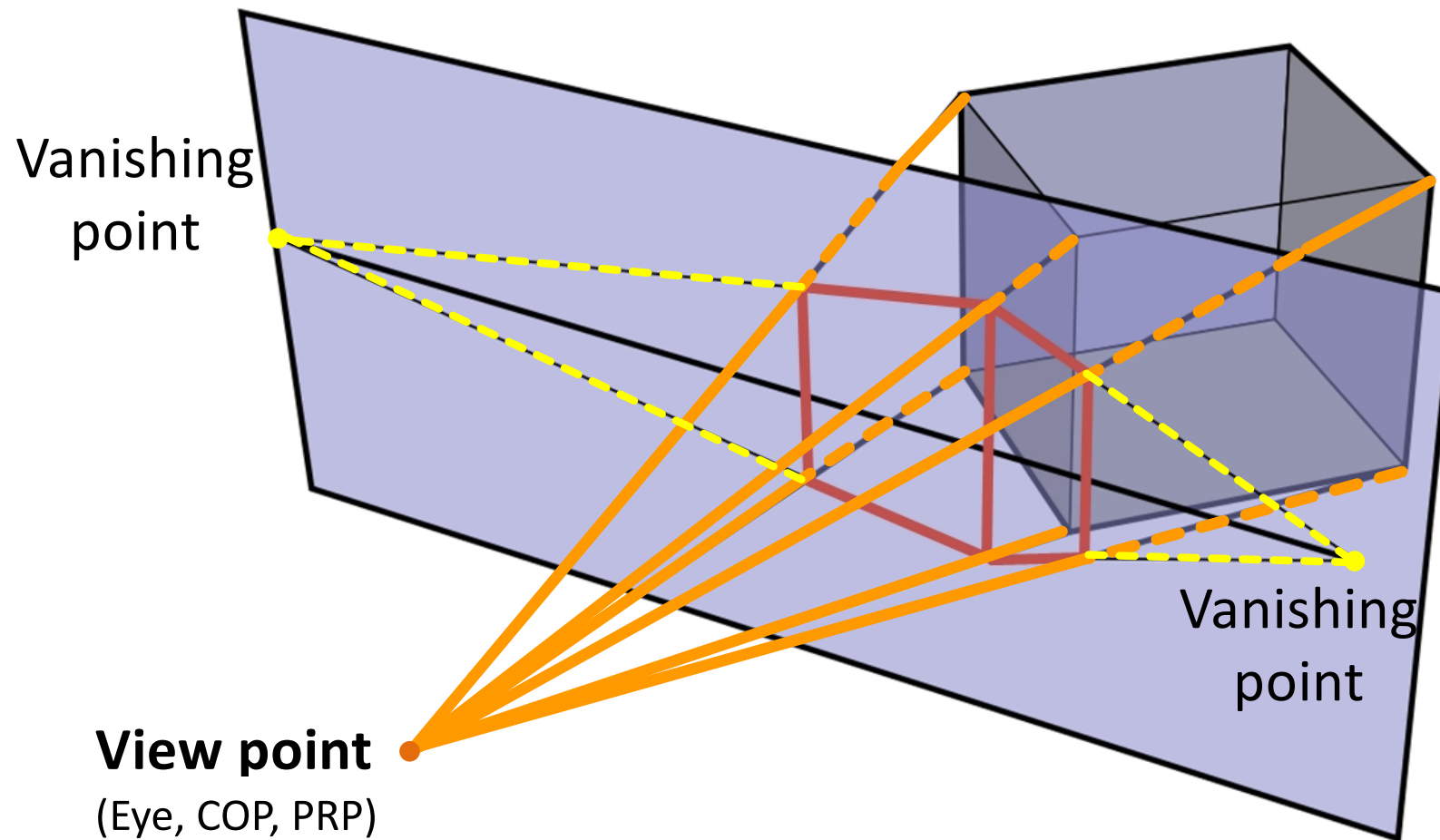
**Two Point Perspective**  
(z, and x-axis vanishing points)



**Three Point Perspective**  
(z, x, and y-axis vanishing points)

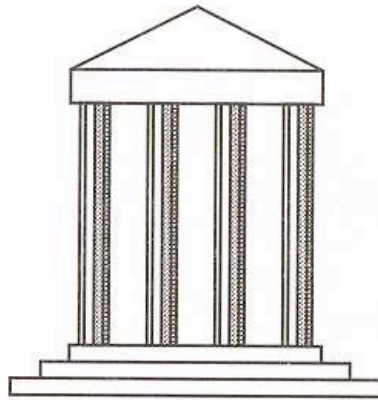
- Note: this is a canonic object
- What happens in case of an arbitrary object?
- Can we have a perspective with 0 vanishing points?

# Viewpoint and vanishing points

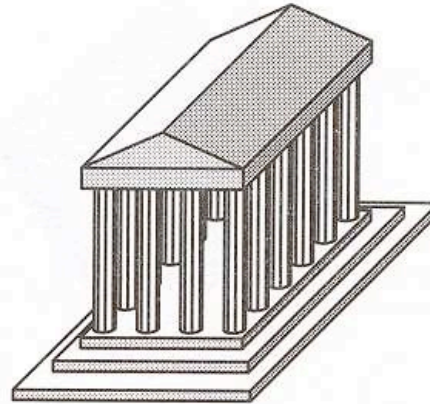


Note two types of pyramidal structure: intersecting the view plane and within the view plane

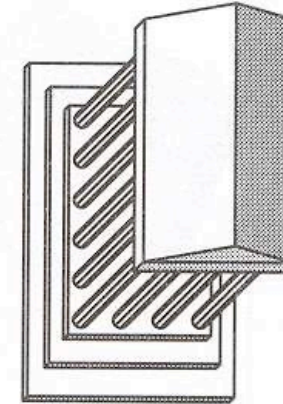
# Examples of some typical projections



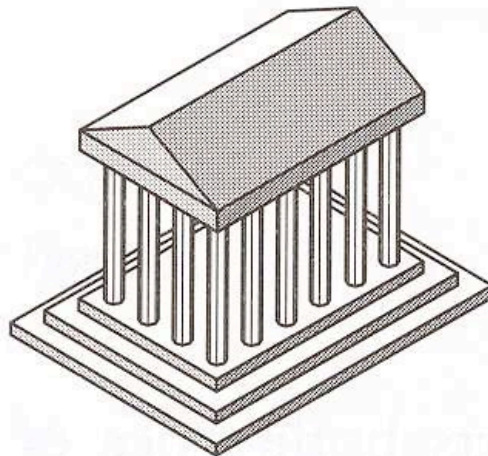
Front elevation



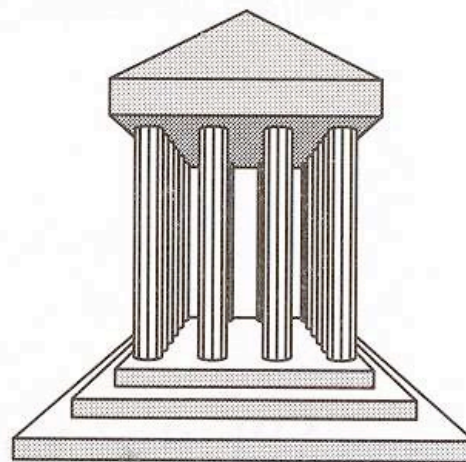
Elevation oblique



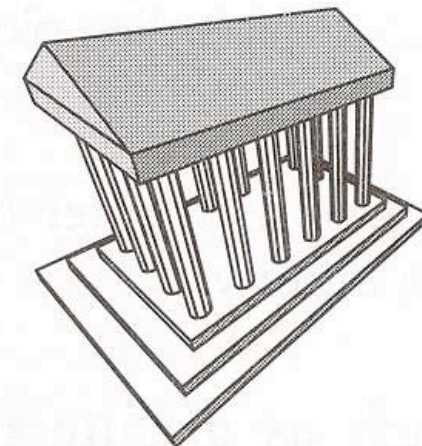
Plan oblique



Isometric



One-point perspective



Three-point perspective

# Perspective versus parallel projection

- Perspective:
  - Visual effect is similar to human visual system
  - Has 'perspective foreshortening'
    - size of object varies inversely with distance from the center of projection.
  - Angles only remain intact for faces parallel to projection plane
  - “Looks” good but not in particular useful for recording the exact shape and distances
- Parallel:
  - Less realistic view because of no foreshortening
  - However, parallel lines remain parallel
  - Angles only remain intact for faces parallel to projection plane

# Synthetic Camera

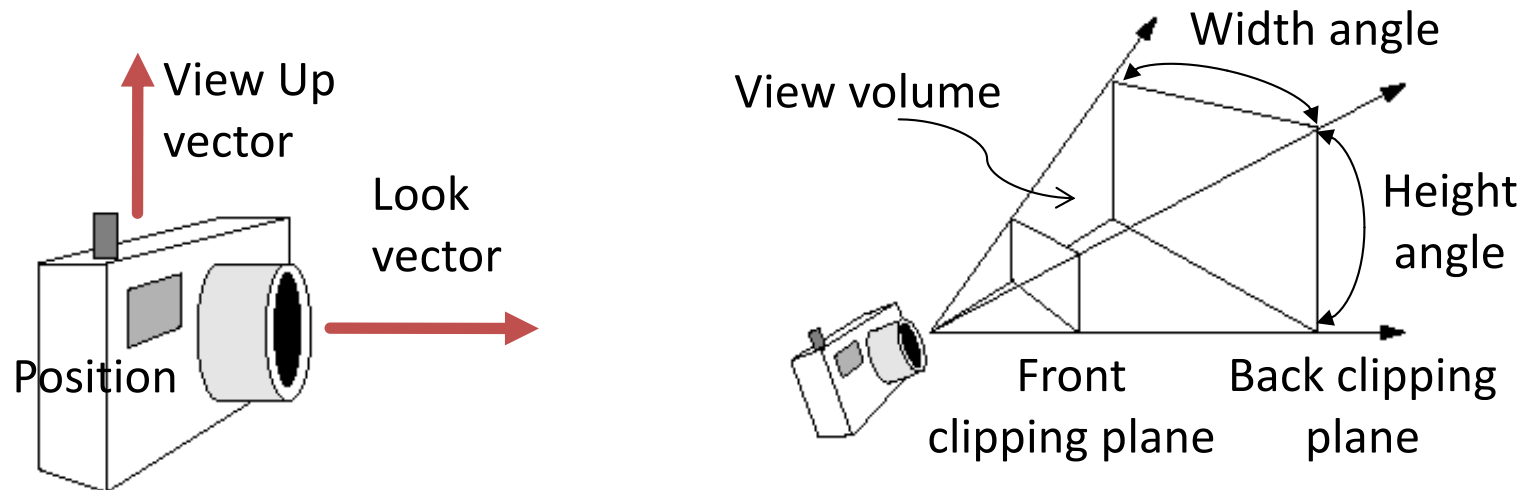


# Synthetic camera model

In order to “take a picture” with our synthetic camera, we need to know

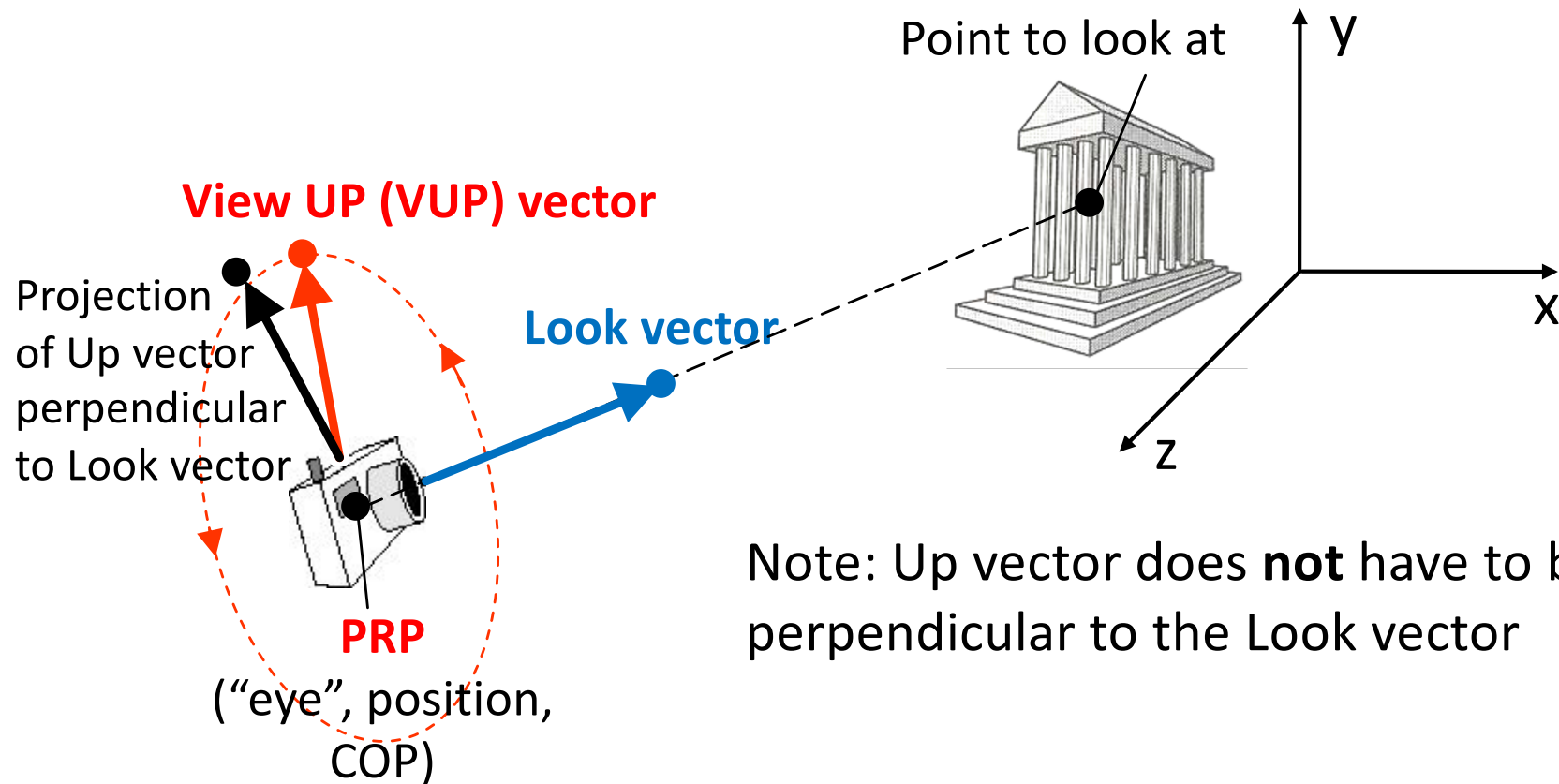
1. **Position** of the camera (from where it's looking; **'eye' vector**)
2. **Orientation** (direction in which the camera is pointing **Look vector** and the rotation around that direction – **View Up vector**)
3. **Field of view** (aspect ratio of the electronic “film”, viewing angles)
4. **Front and back clipping planes**

**Optional:** Focal length (objects at other distances than this are blurred)



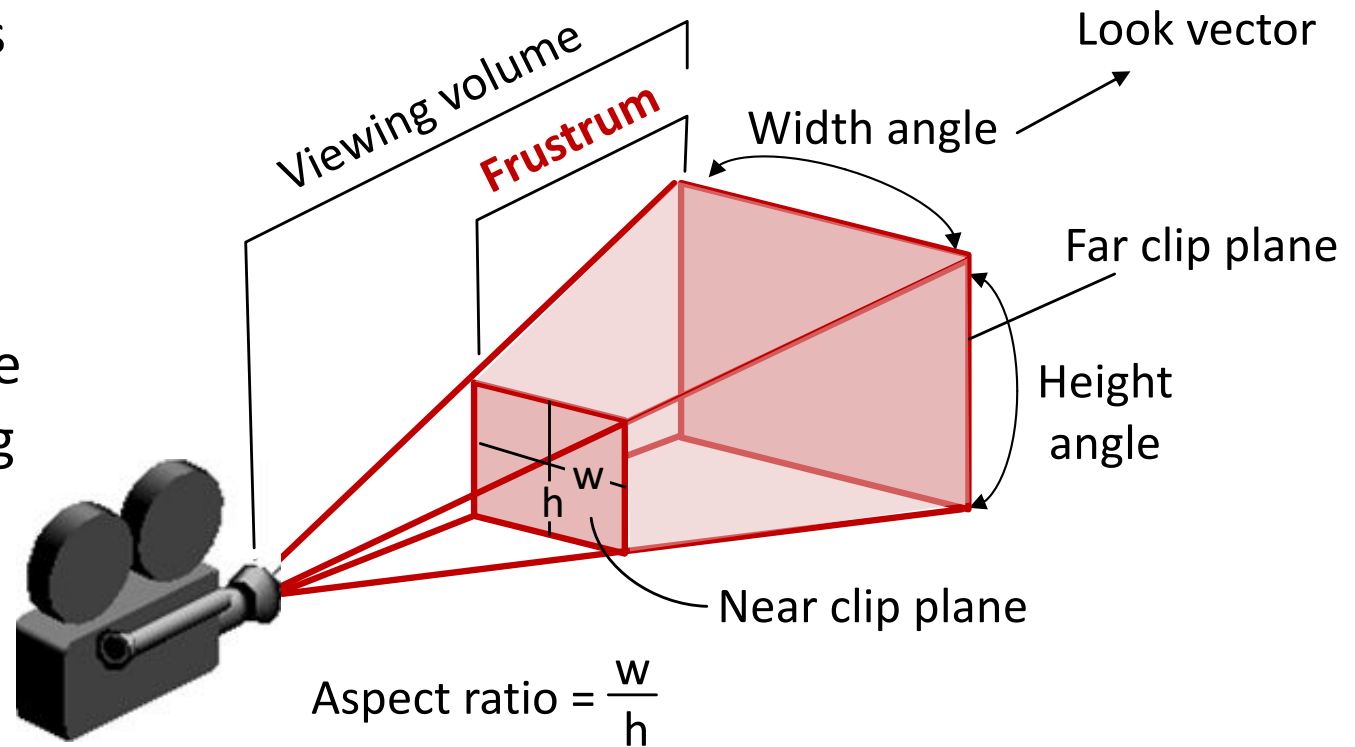
# Position, Look and View Up vectors

- Camera orientation is specified by a point in 3D space to look at (or a direction to look in) and an angle of rotation about this direction. These correspond to the **Look vector** and **View UP (VUP) vector**.



# Aspect Ratio, Viewing Angle, Clipping Planes

- Two viewing angles (width and height)
- Usually, the height angle is specified and the width angle is determined using the aspect ratio



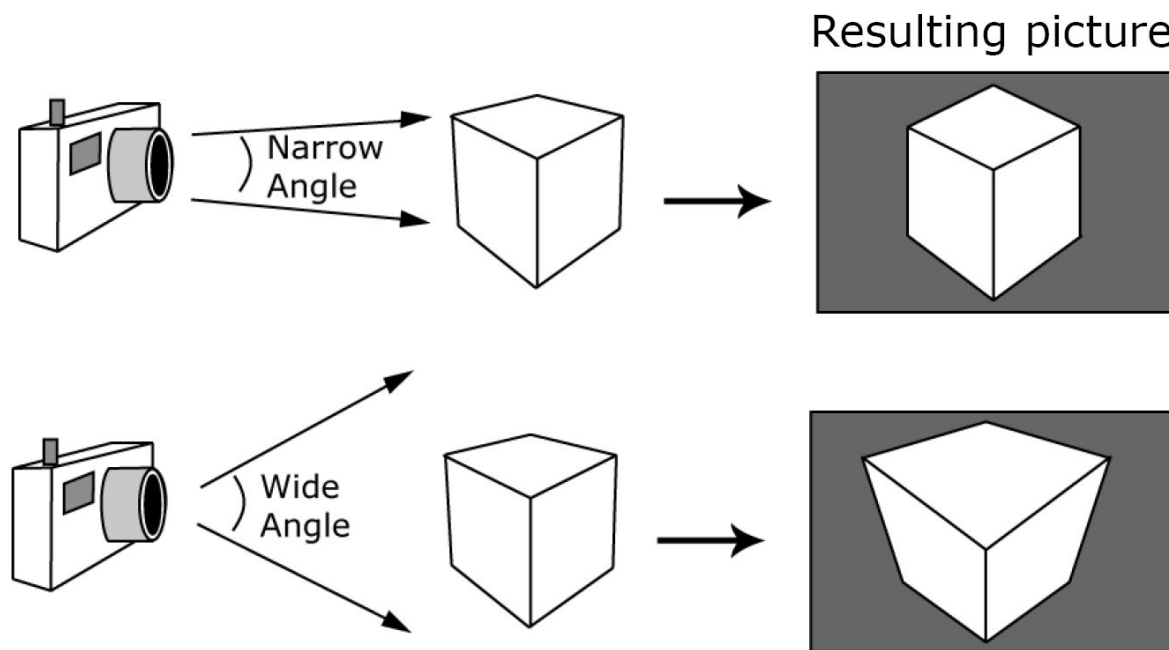
## Frustrum:

- part of the viewing volume between the front and back clipping planes
- in general (perspective projection) this is a truncated pyramid
- reduces to a parallelepiped for parallel projection

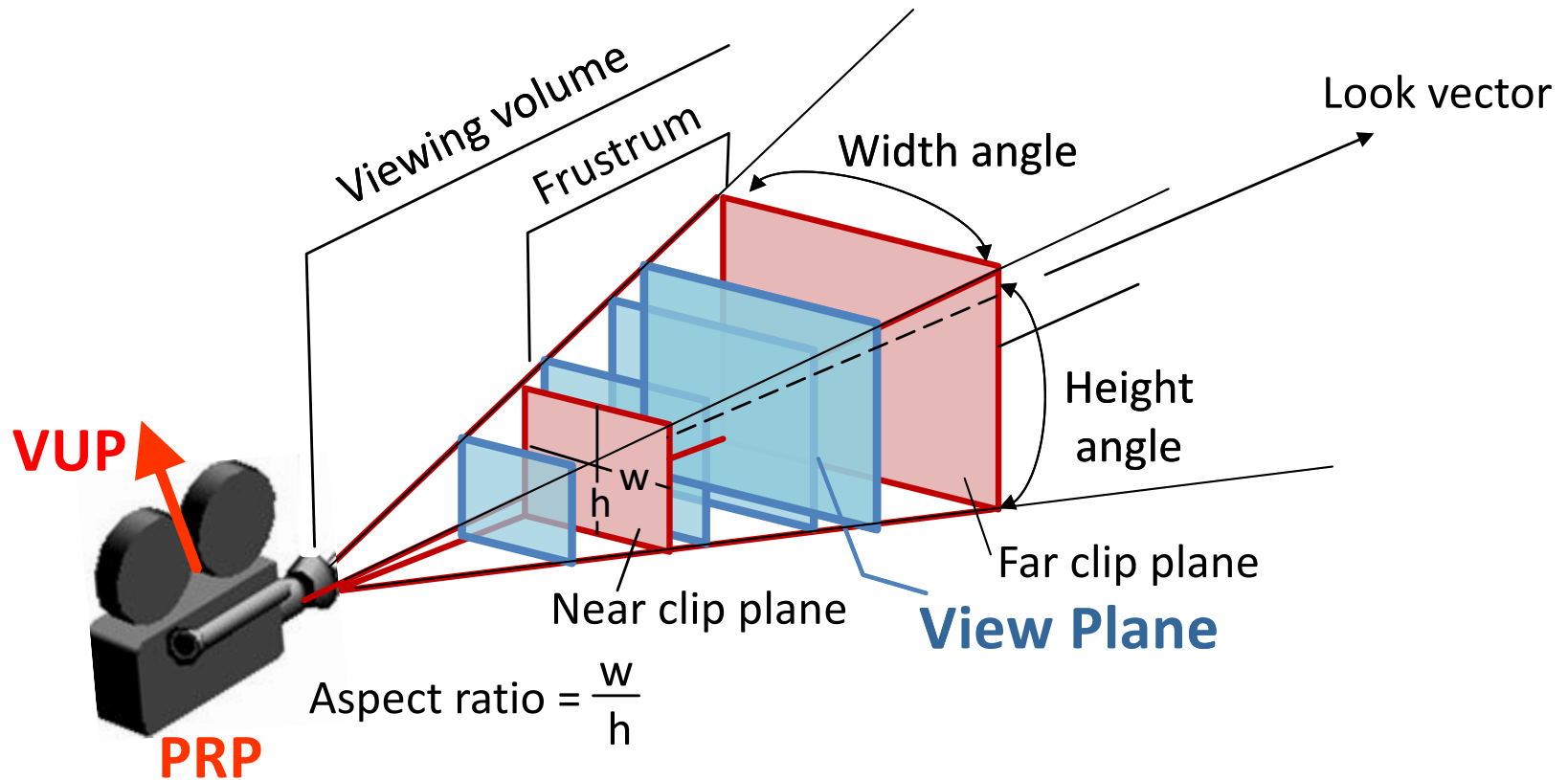


# Viewing angle

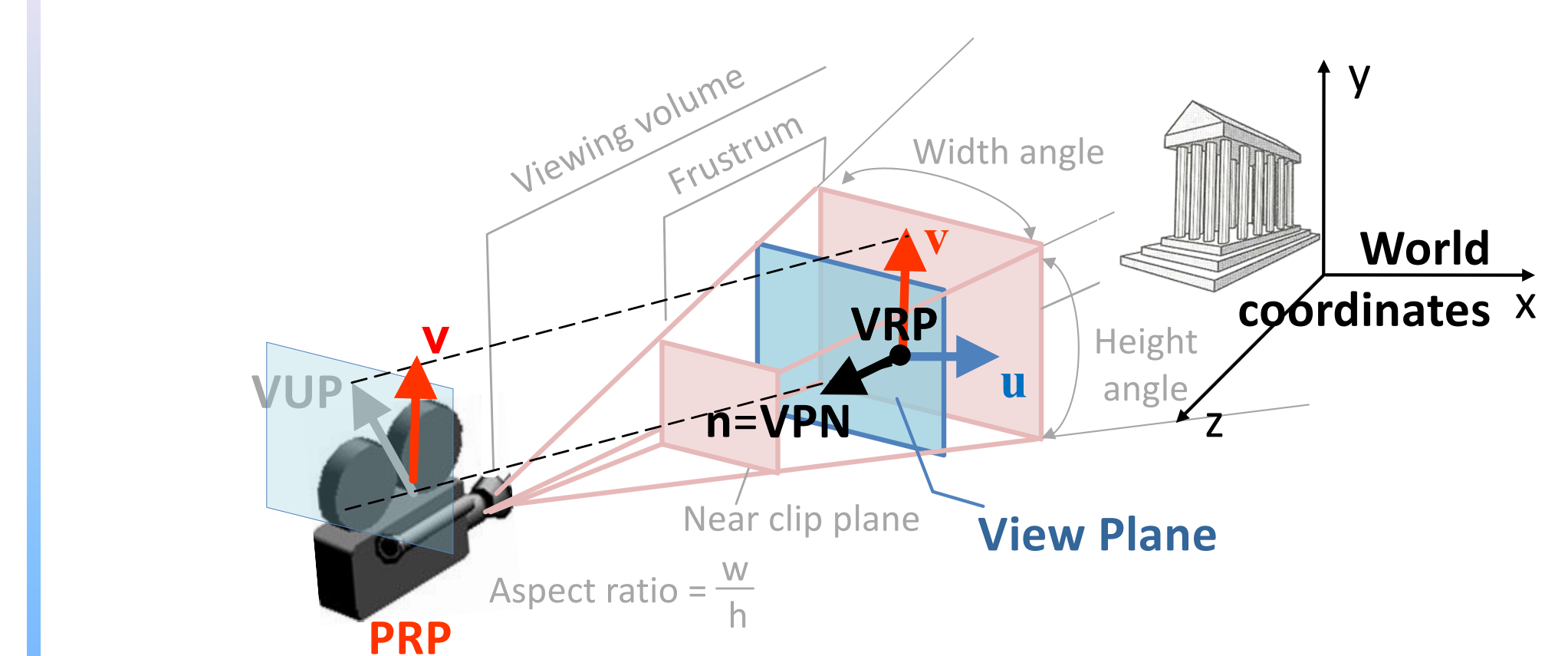
- Viewing angle determines the amount of **perspective distortion** in the resulting picture
  - For zero viewing angle (parallel projection) → no distortion
  - The larger the viewing angle, the larger distortion



# View plane



- The **view plane** can be anywhere with respect to the world objects to be projected: in front of, cut through or behind the objects

[illegible]

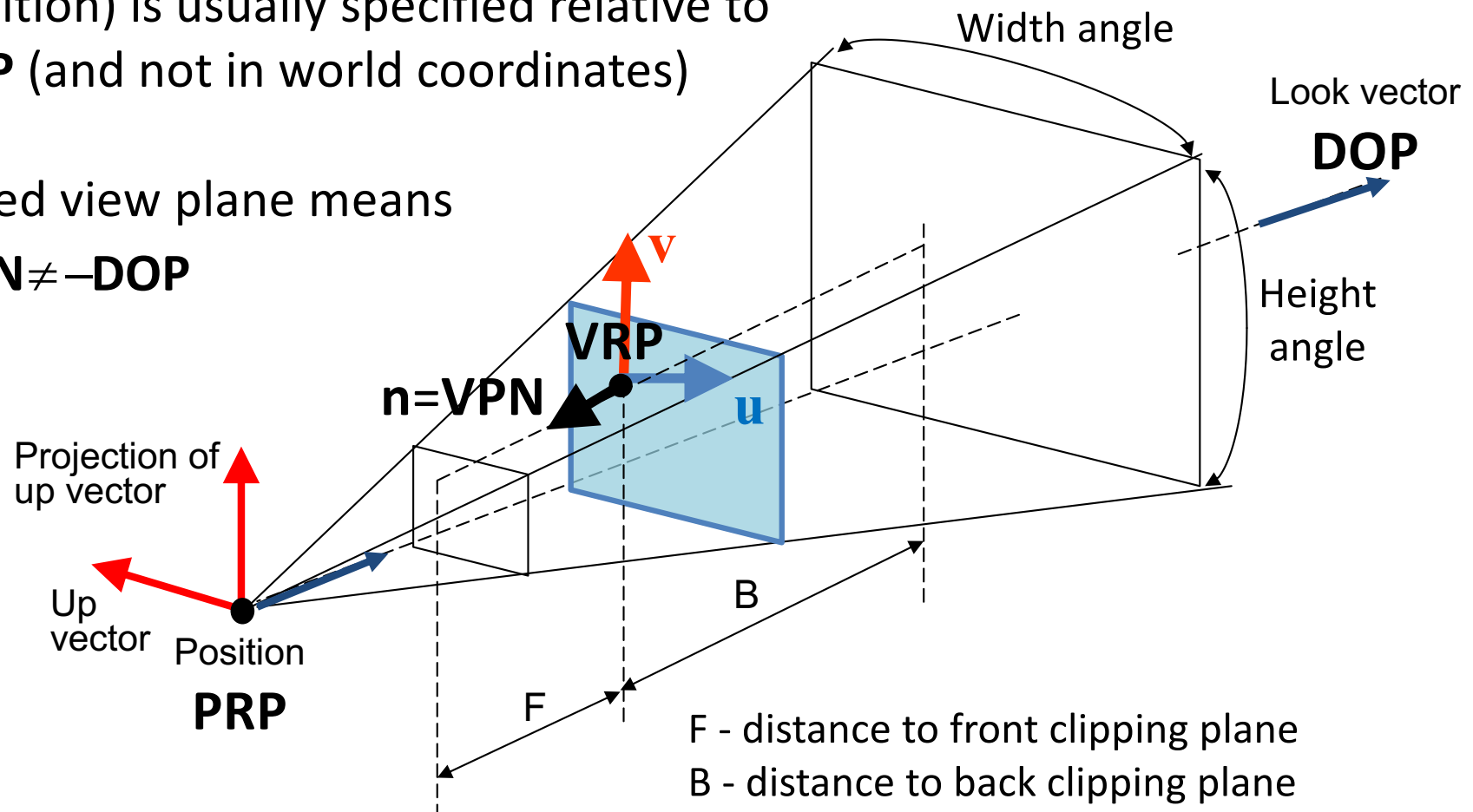
- View Reference Point (VRP) specifies the view plane together with VPN
- View Reference System (VRC) is the right-handed system **u-v-n**
  - **n** is VPN (View Plane Normal)
  - **v** is the projection of the View UP (VUP) vector in the view plane
  - **u** is defined such that **u**, **v** and **n** form a right-handed coordinate system

# Perspective View Volume - Summary

**PRP** – Projection Reference Point  
(i.e., center of projection, camera position) is usually specified relative to **VRP** (and not in world coordinates)

Tilted view plane means

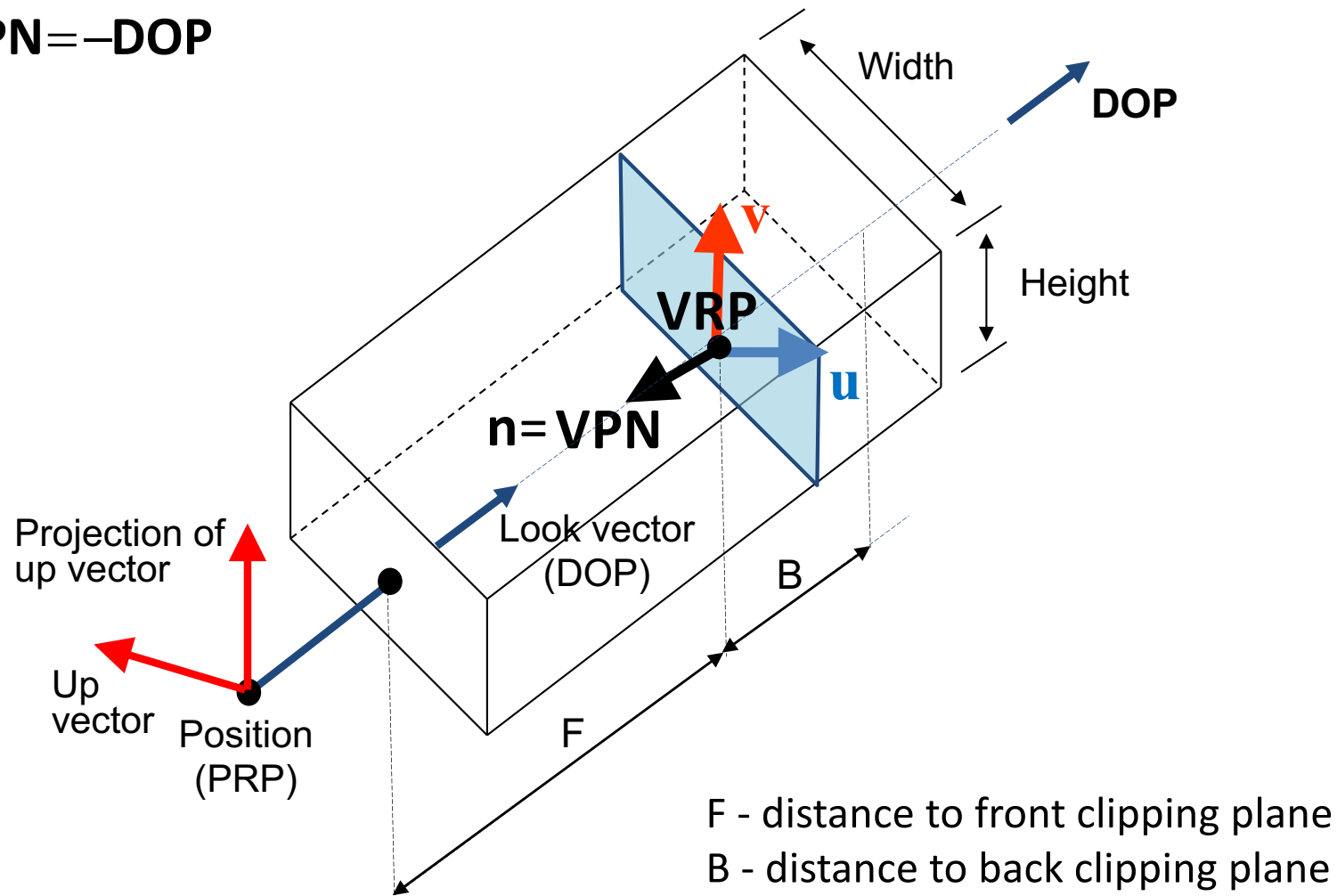
**VPN**  $\neq$  **-DOP**



# View Volume for Parallel Projection (1)

Orthographic case

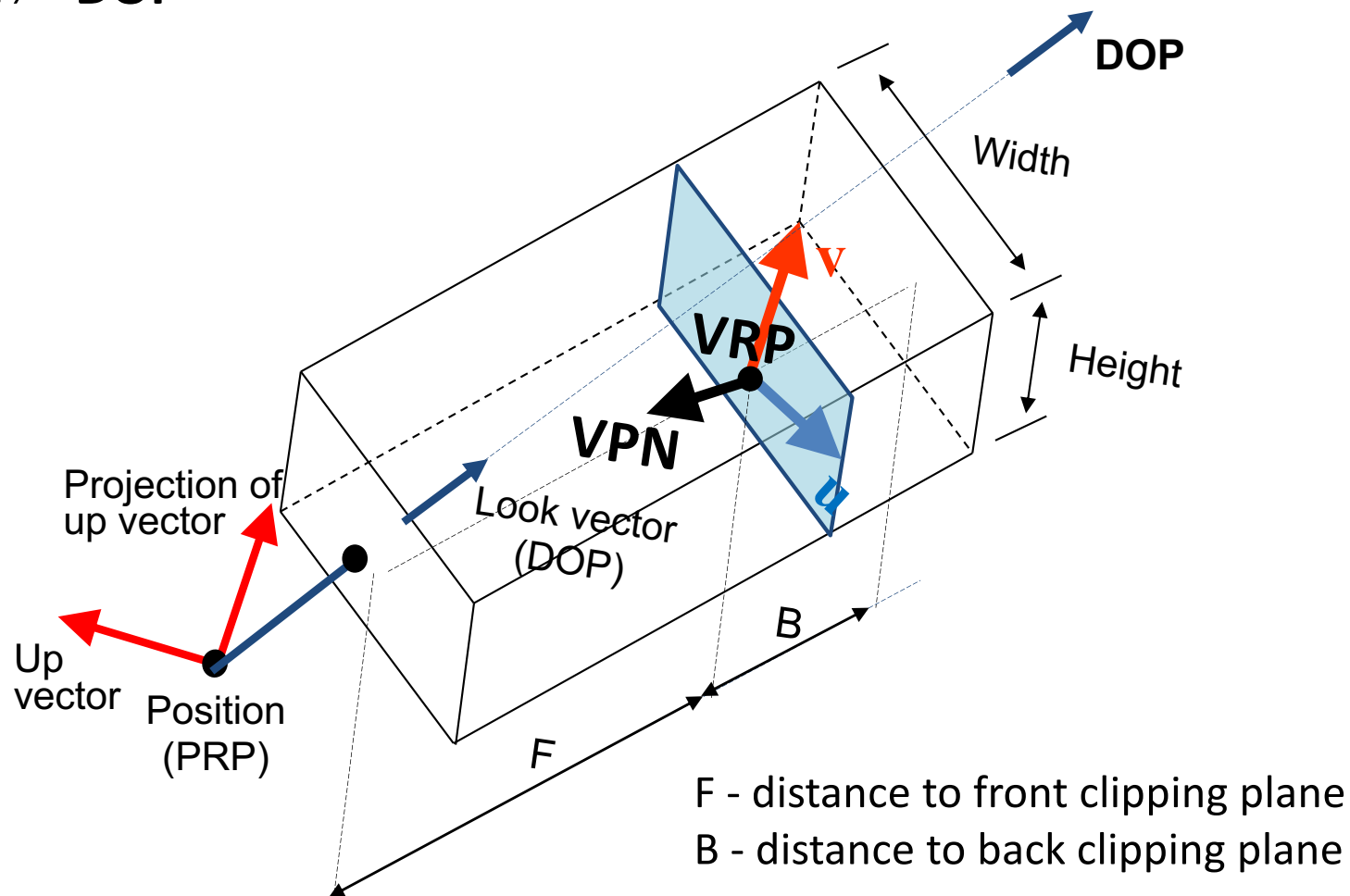
$\text{VPN} = -\text{DOP}$



# View Volume for Parallel Projection (2)

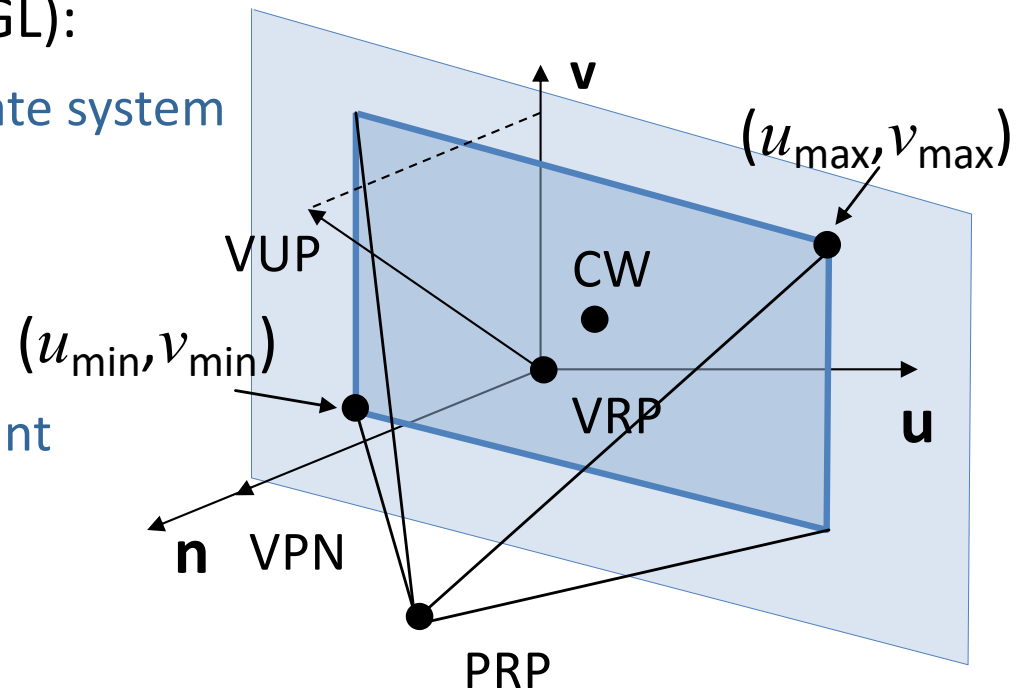
Oblique projection

$VPN \neq -DOP$



# View Reference System vs. World Coordinates

- 3D objects that we are viewing “live” in a **World Coordinate System** – **WCS** (right-handed coordinate system with  $x$ ,  $y$  and  $z$  axes)
- In camera space, we defined **Viewing Reference System** with unit vectors  $\mathbf{u}$ ,  $\mathbf{v}$  and  $\mathbf{n}$  (which is also a right-handed coordinate system)
- Common notation in graphics application program interfaces - API (PHIGS, OpenGL):
  - VRC – view reference coordinate system
  - VRP – view reference point
  - VPN – view plane normal
  - VUP – view up vector
  - PRP – projection reference point
  - CW – center of **window**



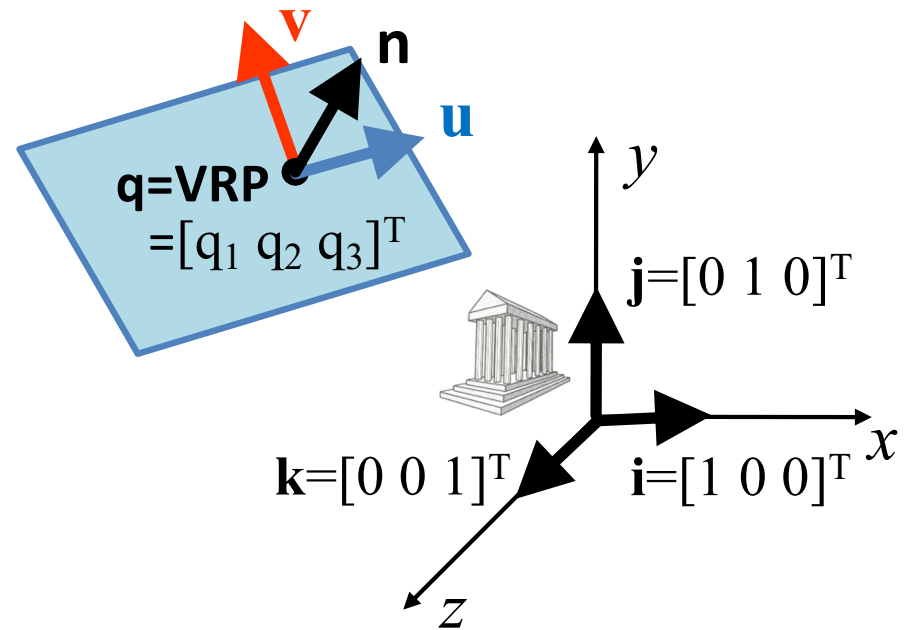
# Transformation from VRC to World Coordinates

VRP in homogeneous coordinates:  $\begin{bmatrix} \mathbf{q} \\ 1 \end{bmatrix}$

We search  $\mathbf{M}$  such that:  $\mathbf{M} \begin{bmatrix} \mathbf{q} \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix}$

Rotation Translation

$$\mathbf{M} = \begin{bmatrix} \mathbf{R} & \mathbf{t} \\ \mathbf{0} & 1 \end{bmatrix}$$



$$\left. \begin{array}{l} \mathbf{R}\mathbf{u} = [1 \ 0 \ 0]^T \\ \mathbf{R}\mathbf{v} = [0 \ 1 \ 0]^T \\ \mathbf{R}\mathbf{n} = [0 \ 0 \ 1]^T \end{array} \right\} \mathbf{R}[\mathbf{u} \ \mathbf{v} \ \mathbf{n}] = \mathbf{I} \Rightarrow \mathbf{R} = [\mathbf{u} \ \mathbf{v} \ \mathbf{n}]^T = \begin{bmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ n_1 & n_2 & n_3 \end{bmatrix}$$

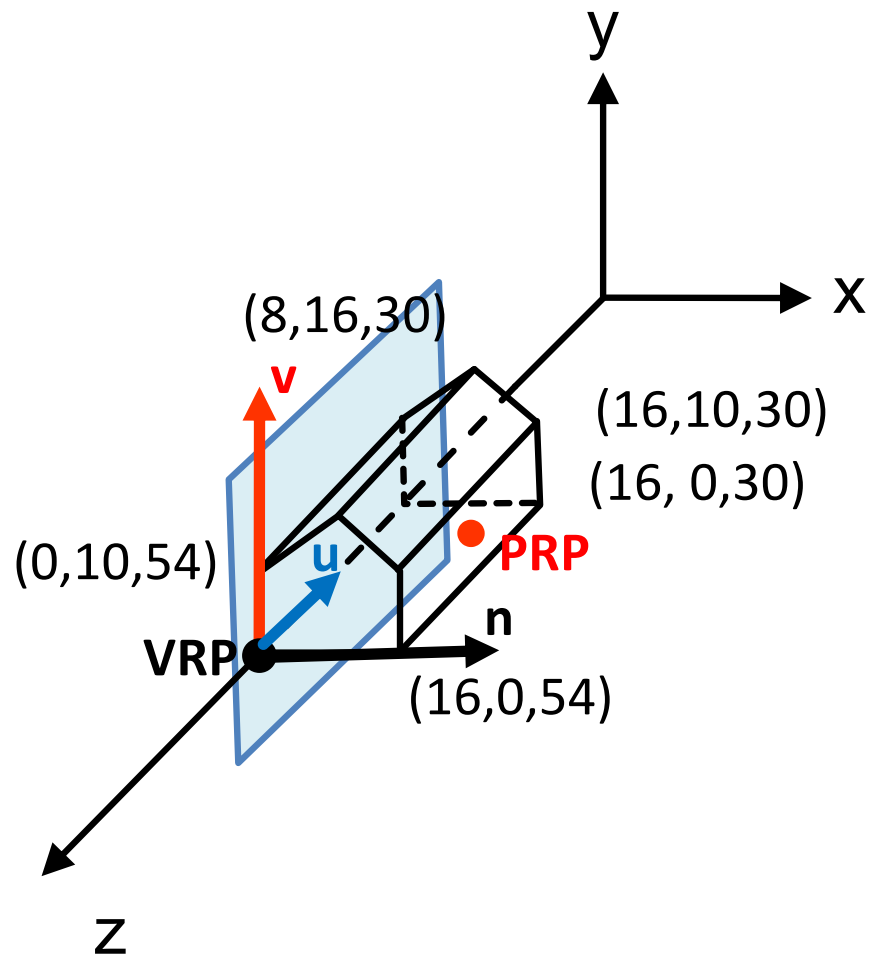
$$\begin{bmatrix} \mathbf{R} & \mathbf{t} \\ \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} \mathbf{q} \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} \Rightarrow \mathbf{R}\mathbf{q} + \mathbf{t} = \mathbf{0} \Rightarrow \mathbf{t} = -\mathbf{R}\mathbf{q} = -\left[ \sum_i u_i q_i \quad \sum_i v_i q_i \quad \sum_i n_i q_i \right]^T$$



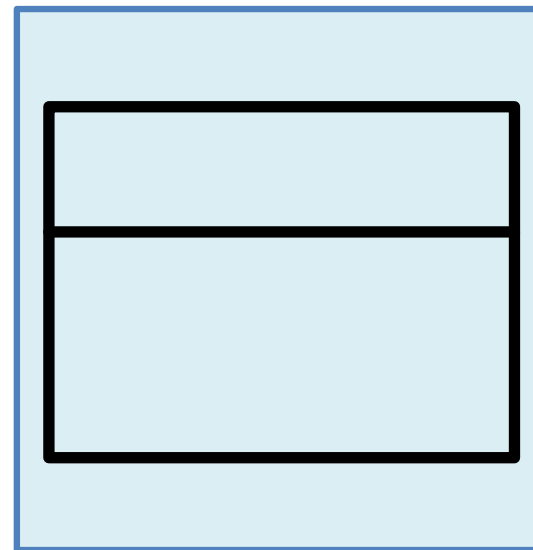
# View specification examples



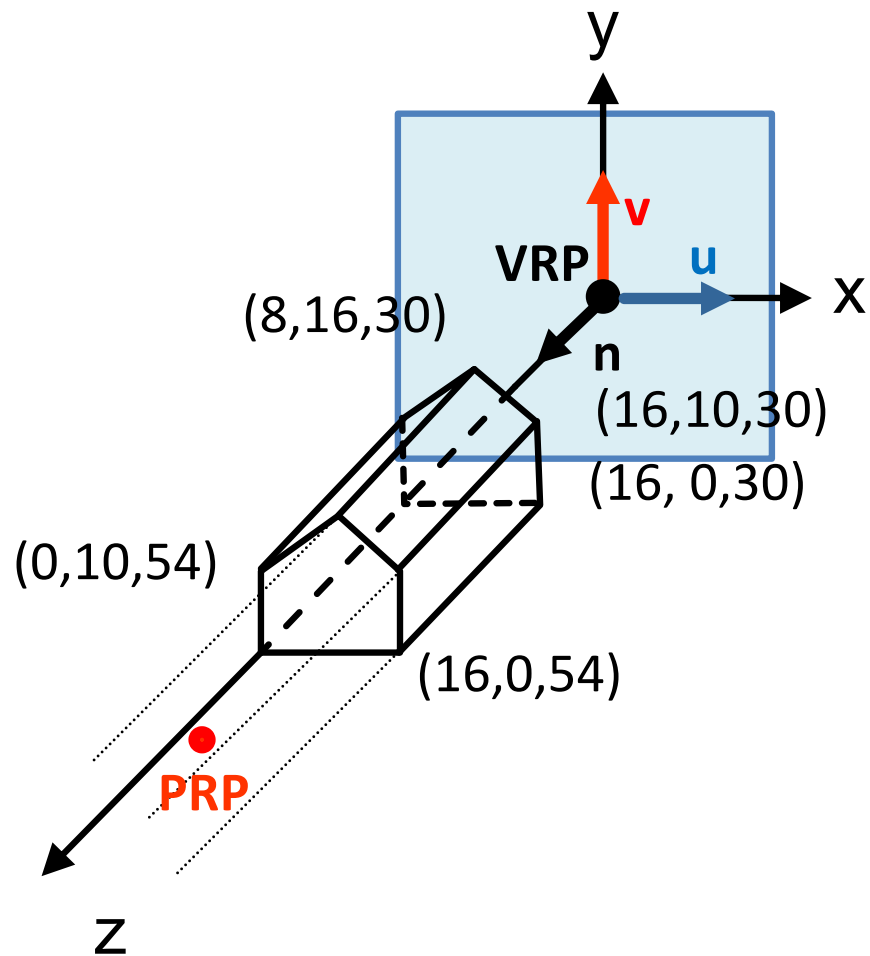
# Specifying arbitrary view in 3D: Examples



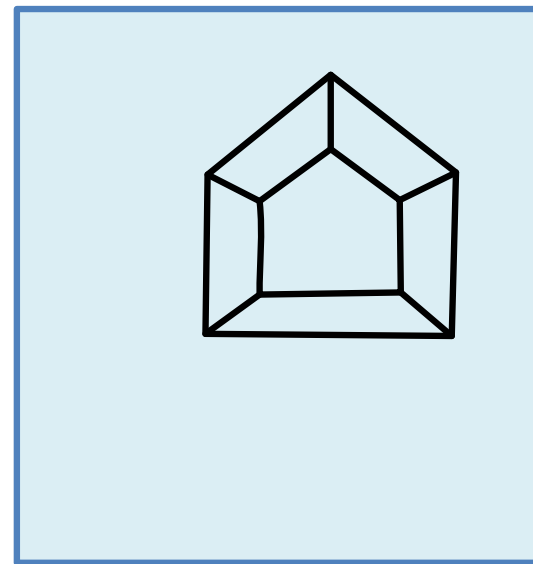
VRP (WC): (0,0,54)  
VPN (WC): (1,0,0)  
VUP (WC): (0,1,0)  
PRP (VRC): (12,8,16)  
Window (VRC): (-1,25,-5,21)  
Projection type: parallel



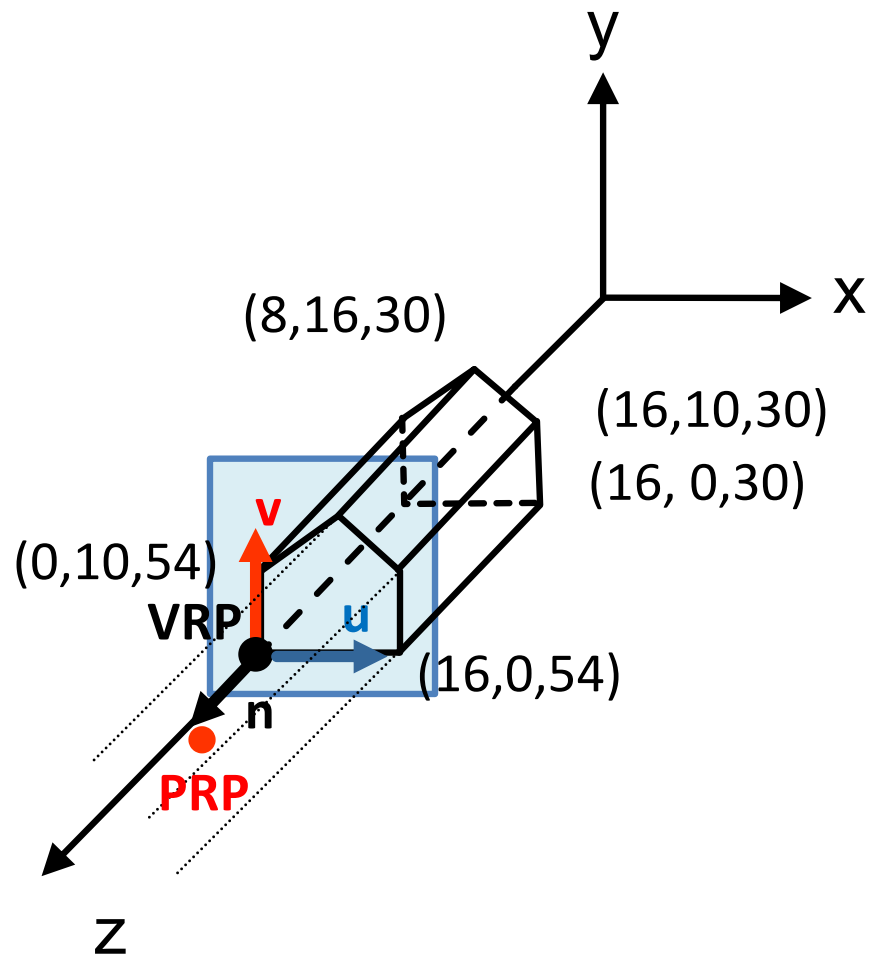
# Specifying arbitrary view in 3D: Examples



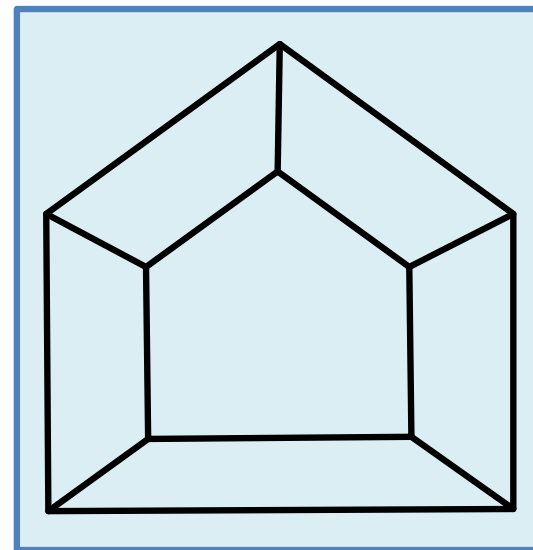
VRP (WC): (0,0,0)  
VPN (WC): (0,0,1)  
VUP (WC): (0,1,0)  
PRP (VRC): (8,6,84)  
Window (VRC): (-50,50,-50,50)  
Projection type: perspective



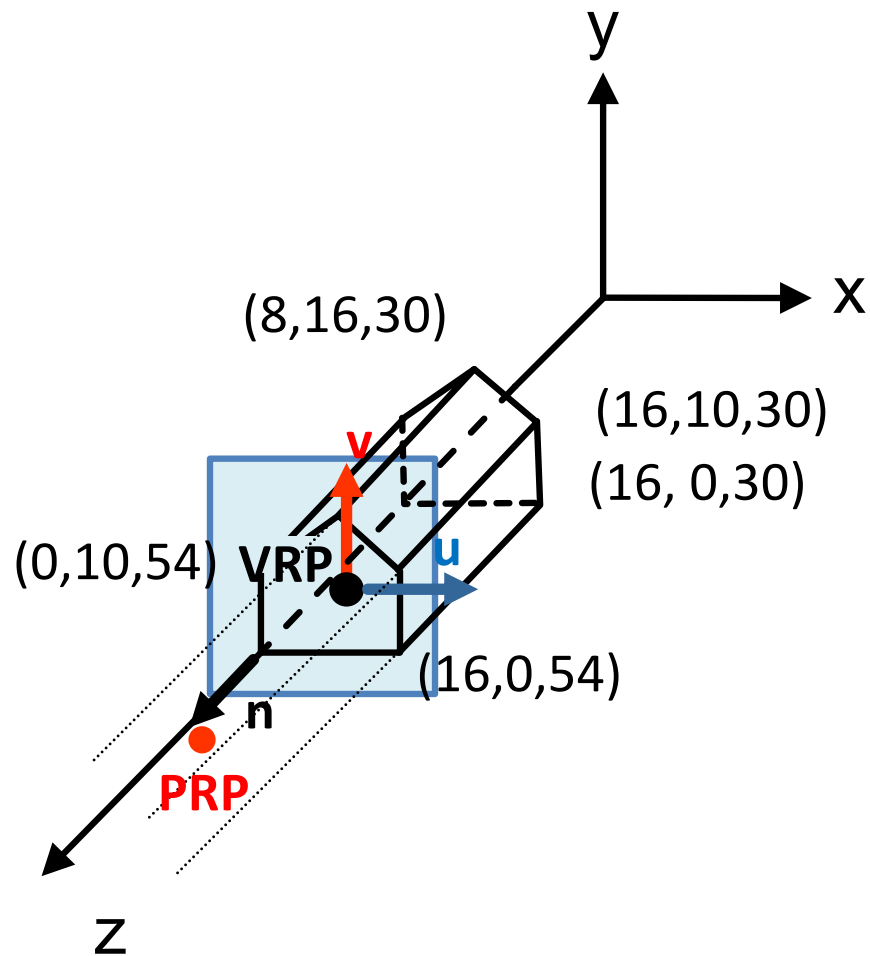
# Specifying arbitrary view in 3D: Examples



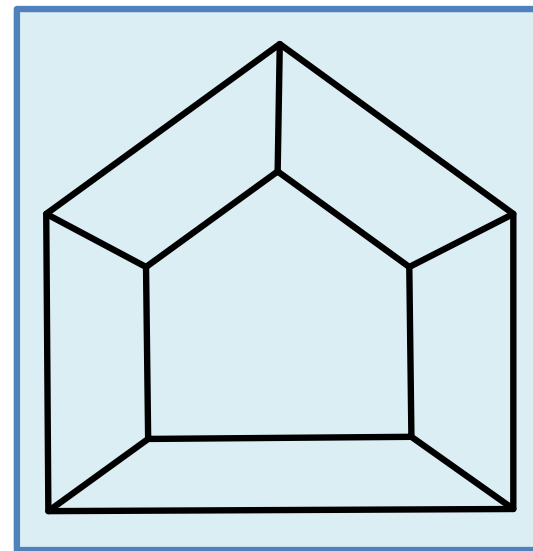
VRP (WC): (0,0,54)  
VPN (WC): (0,0,1)  
VUP (WC): (0,1,0)  
PRP (VRC): (8,6,30)  
Window (VRC): (-1,17,-1,17)  
Projection type: perspective



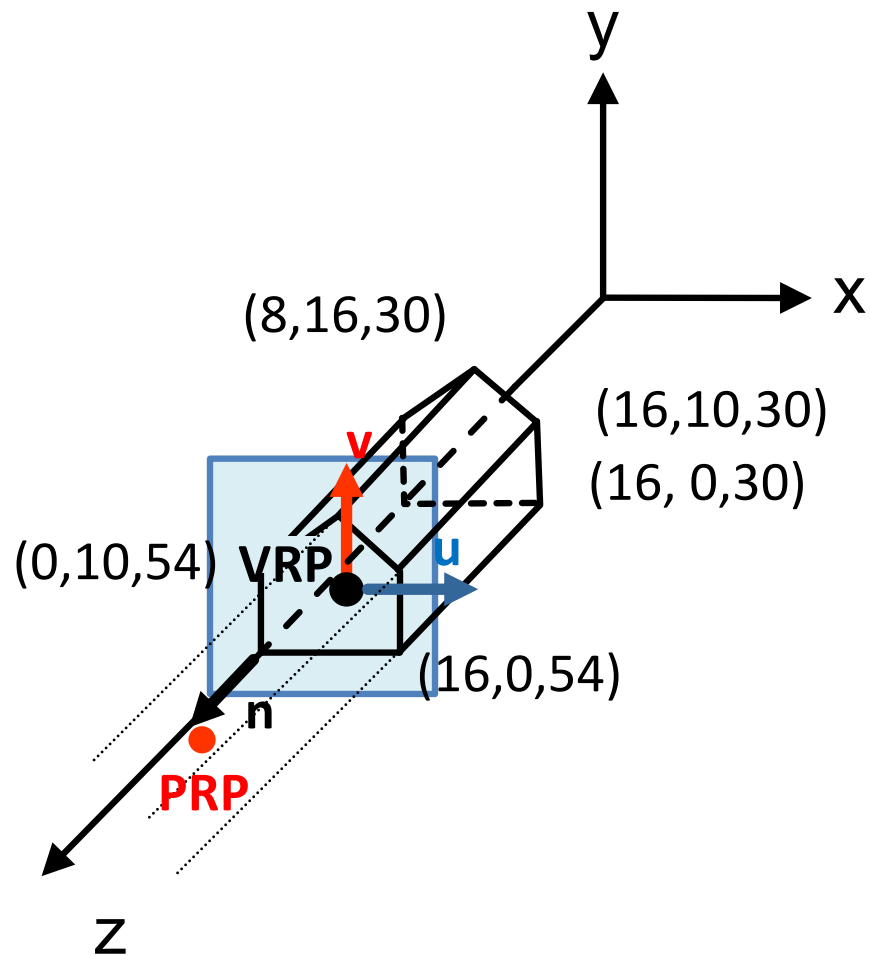
# Specifying arbitrary view in 3D: Examples



VRP (WC):  $(8,6,54)$   
VPN (WC):  $(0,0,1)$   
VUP (WC):  $(0,1,0)$   
PRP (VRC):  $(0,0,30)$   
Window (VRC):  $(-9,9,-7,11)$   
Projection type: perspective

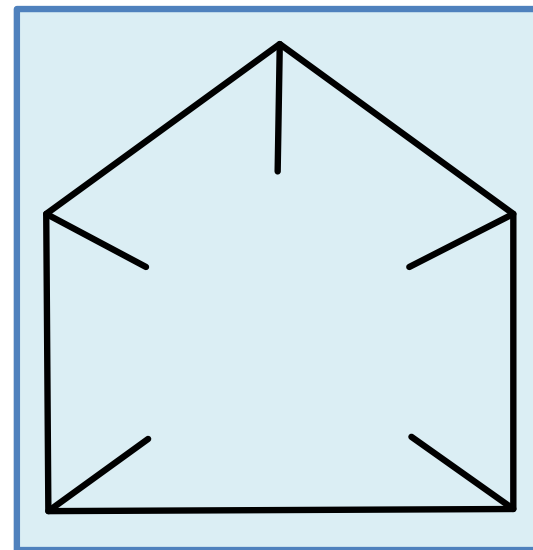


# Specifying arbitrary view in 3D: Examples

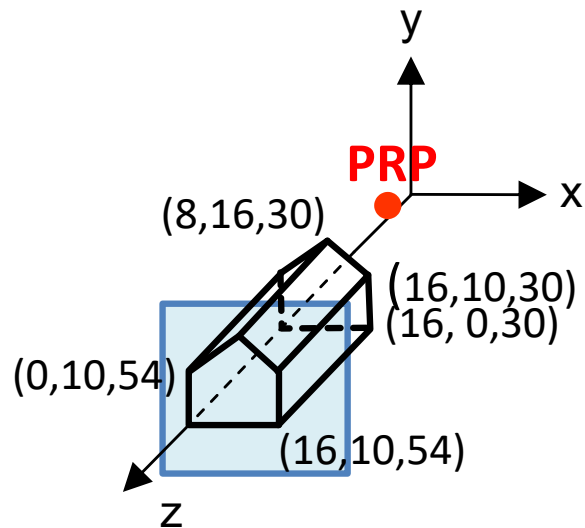


VRP (WC): (8,6,54)  
VPN (WC): (0,0,1)  
VUP (WC): (0,1,0)  
PRP (VRC): (0,0,30)  
Window (VRC): (-9,9,-7,11)  
Projection type: perspective

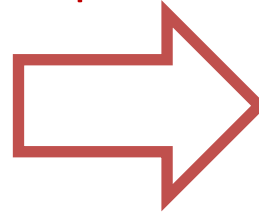
**Back clipping plane** at  $z=31$



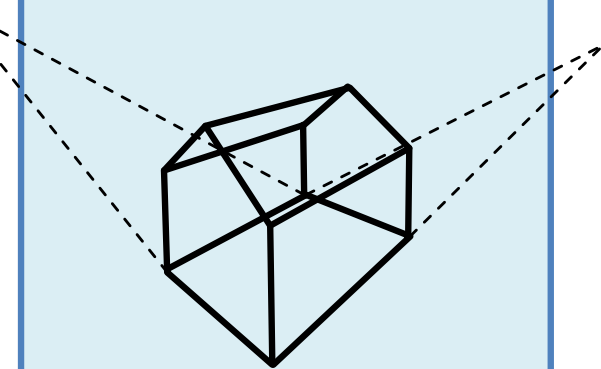
# Specifying arbitrary view in 3D: Examples



Which parameters?

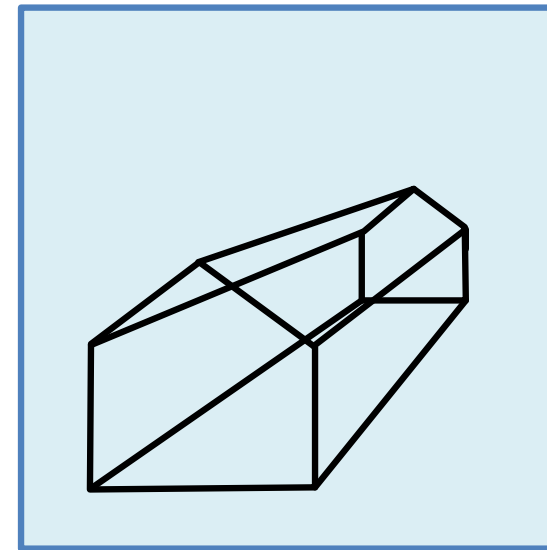
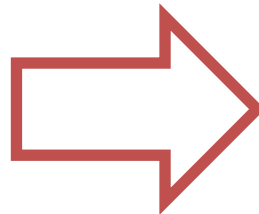


desired



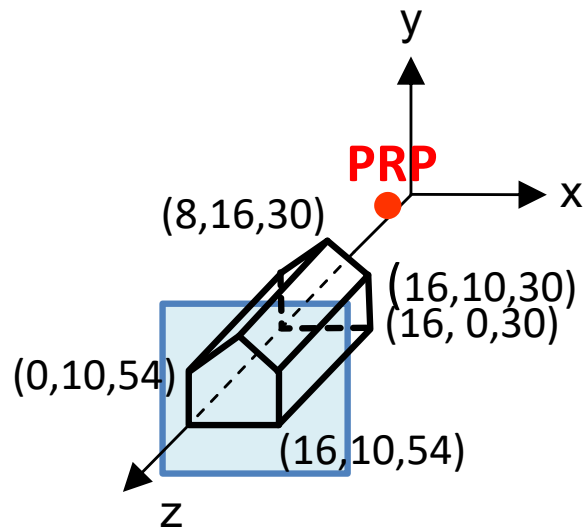
Try the following:

VRP (WC): (16,0,54)  
VPN (WC): (0,0,1)  
VUP (WC): (0,1,0)  
PRP (VRC): (20,25,20)  
Window (VRC): (-20,20,-5,35)  
Projection type: perspective

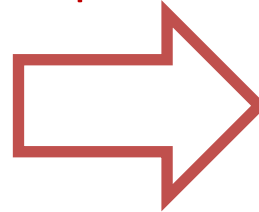


Similar, but what is wrong?

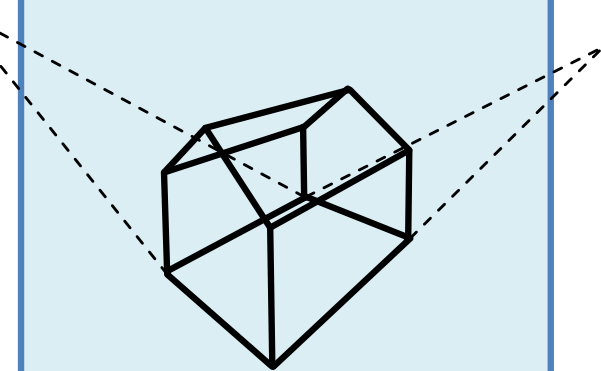
# Specifying arbitrary view in 3D: Examples



Which parameters?

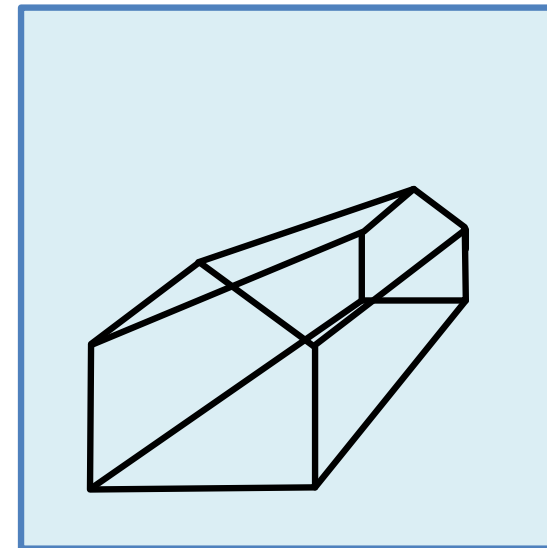
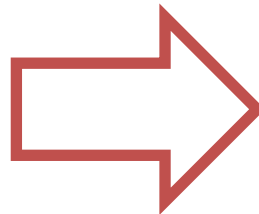


desired



Try the following:

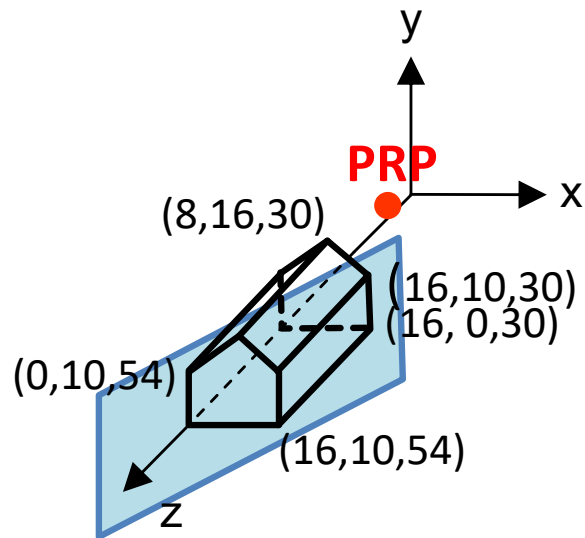
VRP (WC): (16,0,54)  
VPN (WC): (0,0,1)  
VUP (WC): (0,1,0)  
PRP (VRC): (20,25,20)  
Window (VRC): (-20,20,-5,35)  
Projection type: perspective



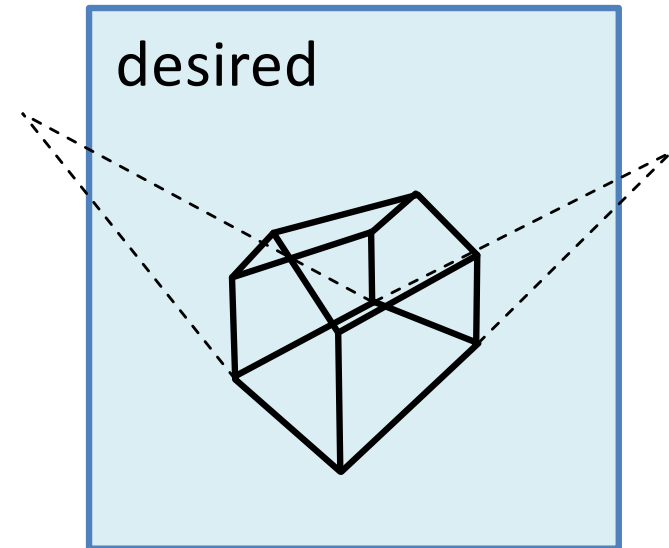
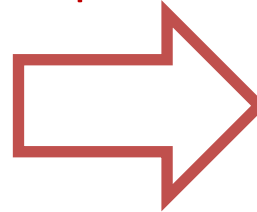
**Need to reorient the view plane!**



## Specifying arbitrary view in 3D: Examples



## Which parameters?



In this case the view plane shouldn't be parallel to x-y plane!

VRP (WC): (16,0,54)

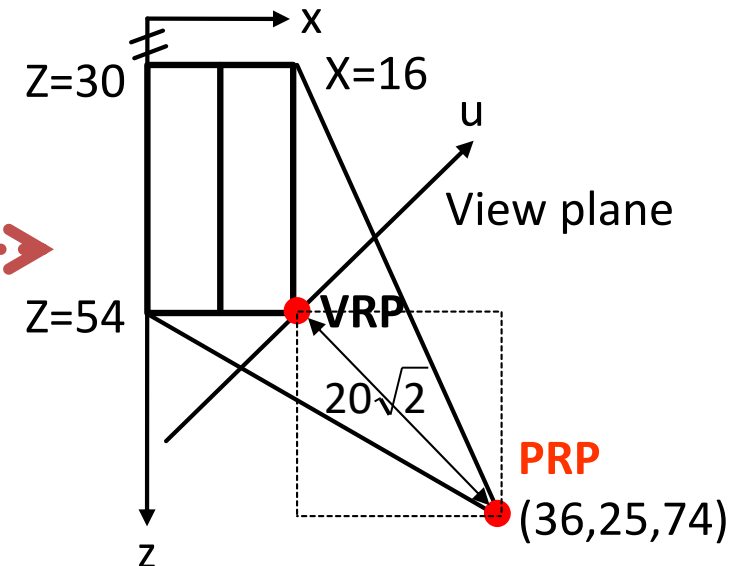
VPN (WC): (1,0,1)

VUP (WC):  $(0,1,0)$

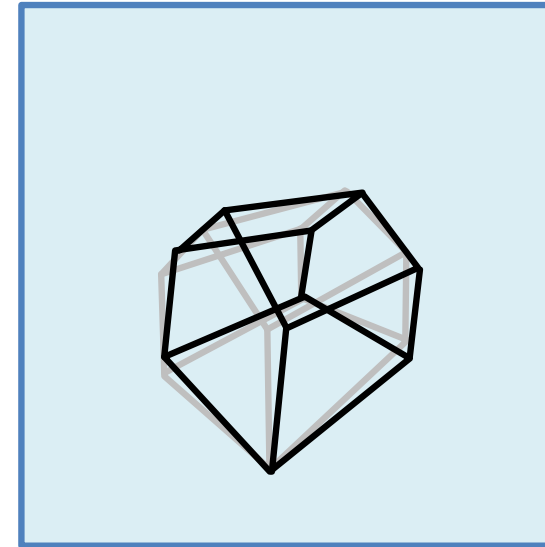
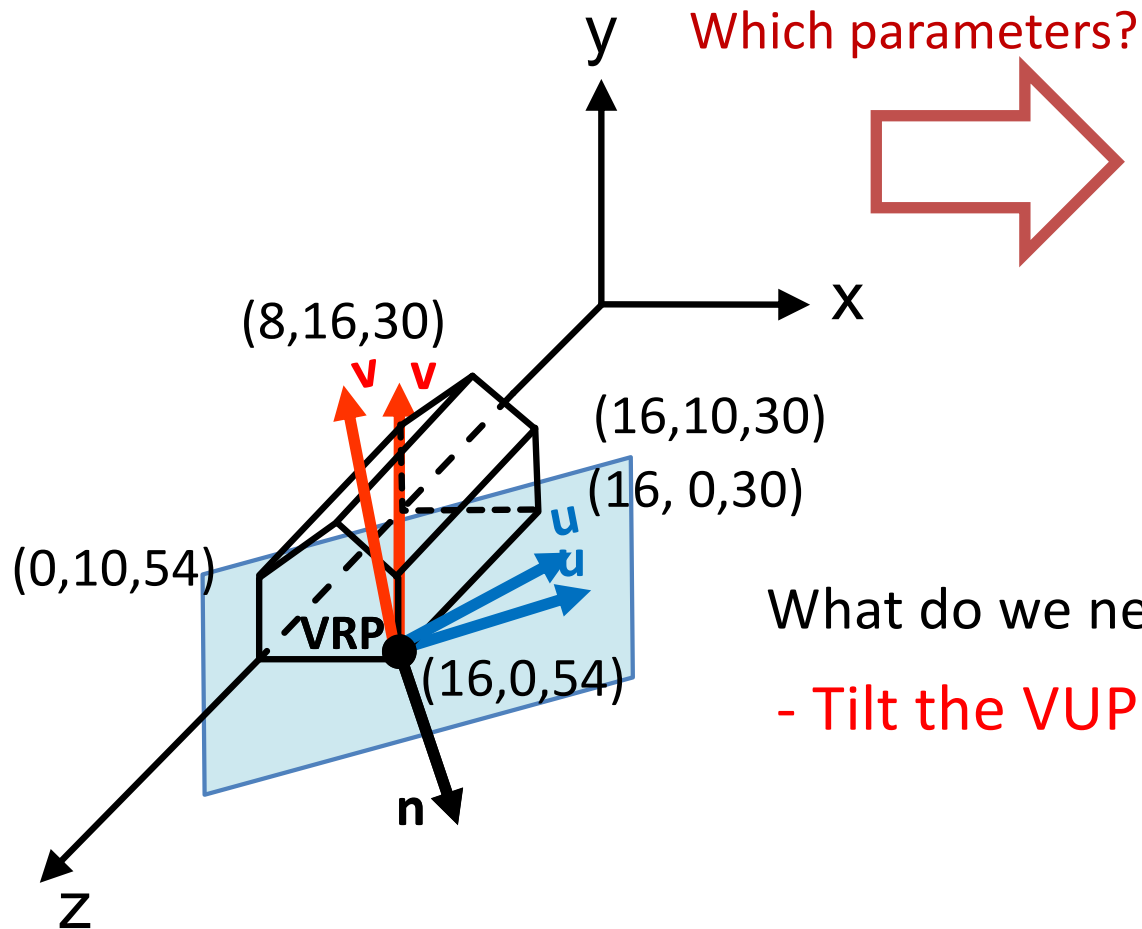
PRP (VRC):  $(0, 25, 20\sqrt{2})$ 

Window (VRC):  $(-20, 20, -5, 35)$

Projection type: perspective



# Specifying arbitrary view in 3D: Examples



What do we need to change to get this view?

- Tilt the VUP vector

# Summary

- We introduced main concepts of viewing in 3D
  - Types of projections
  - Synthetic camera model: important parameters
- Next lesson
  - Mathematics of projections: projection matrices
  - Practical viewing process: transforming arbitrary view volume into canonical view volume