Digital Image Processing

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Image transforms

Introduction to image transforms

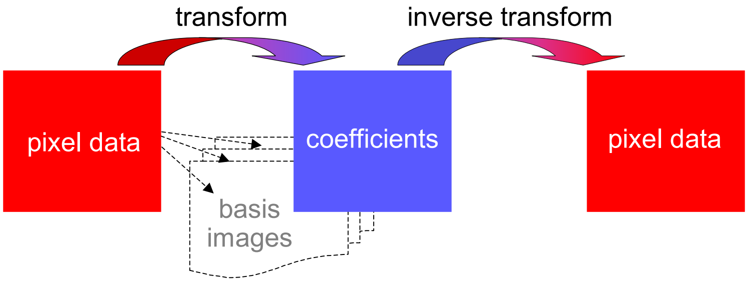

Image transform decomposes an image into **basis images**. Coefficients can be seen as weighting factors for those different "image components"

Important for compression, restoration and analysis. **For example:**

- **Compression**: define image transform such to "pack" the essential information about the image into **as few big coefficients as possible**
- **Restoration**: define image transform such that **biggest image coefficients indicate image edges**
- Other requirements can be defined for specific applications/problems

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Projections onto basis images

$$b(x,y) = \sum_{i=0}^{MN-1} a_i p_i(x,y) \quad x = 0 \ldots M-1,\ y = 0 \ldots N-1$$

$$a_i = \langle B, P_i \rangle$$

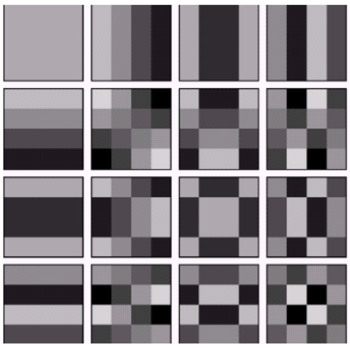
$$\langle B, P \rangle = \sum_{m=0}^{M-1} \sum_{n-0}^{N-1} b(m,n) p^*(m,n)$$ inner product

The transform coefficient a_i is simply the inner product of the i-th basis image with the given image.

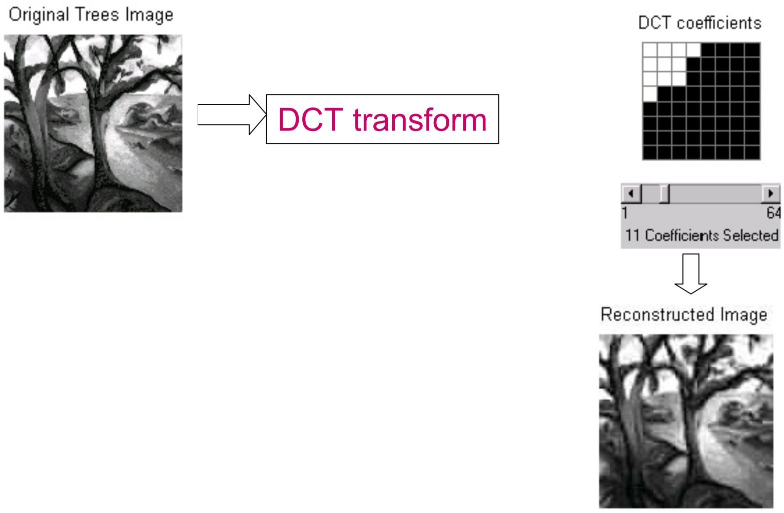
It is also called projection of the image on the i-th basis image

Discrete Cosine Transform

- The cosine transform has excellent energy compaction for highly correlated data
- Often used in compression (JPEG)



Application of DCT im image compression

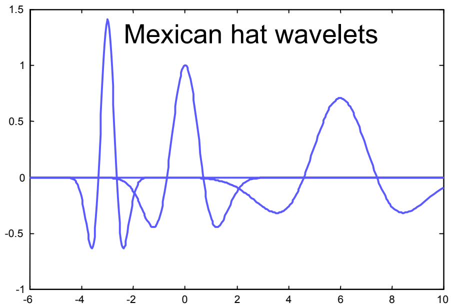


Wavelet transform

Wavelets - localized waves

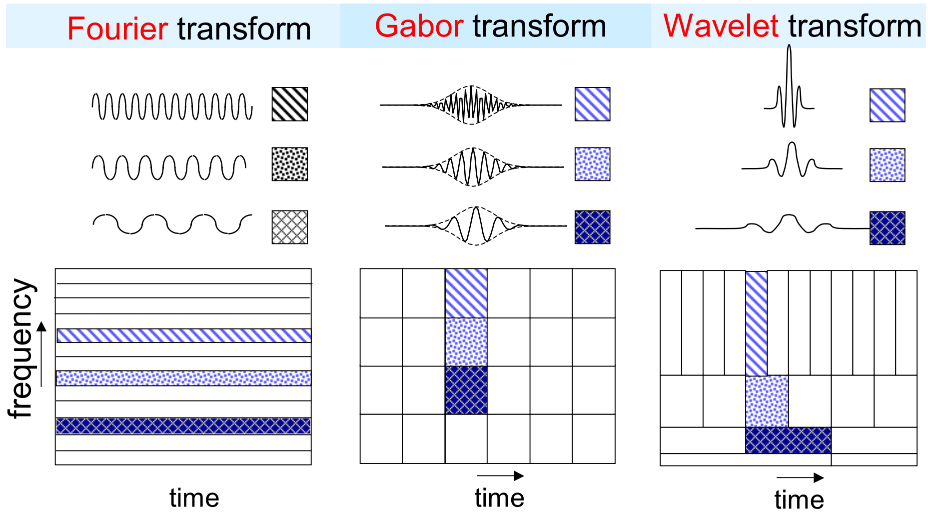
$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right)$$

Wavelet family:
shifts and dilations of the
mother wavelet $\psi(t)$



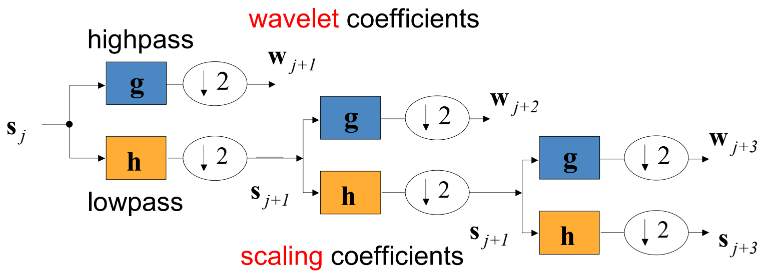
- Continuous wavelet transform: correlate signal with wavelets to reveal structures of different sizes
- Main idea: **analyze according to scale!**
  (see the forest and the trees!)

Wavelet analysis versus Fourier analysis



Discrete Wavelet Transform (DWT)

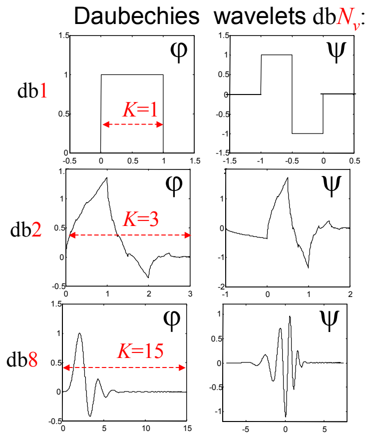
DWT algorithm: a filter bank iterated on the lowpass output

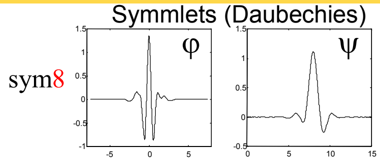


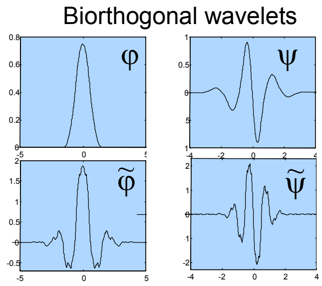
Choosing a wavelet: N_v, support size K, symmetry

N_v - number of vanishing moments: $\int_{-\infty}^{\infty} t^k \psi(t) dt = 0, \quad 0 \le k \le N_v - 1$

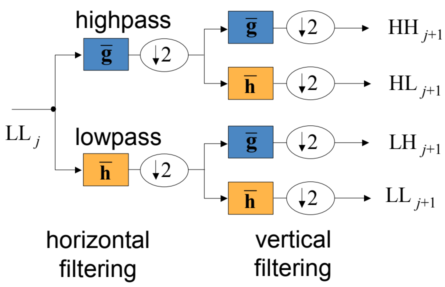
A tradeoff: $K \geq 2N_v$-1

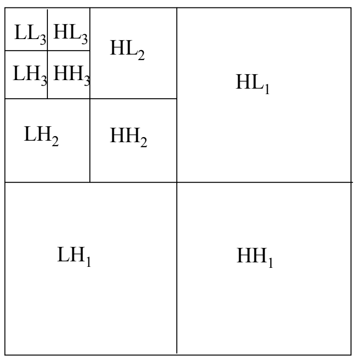




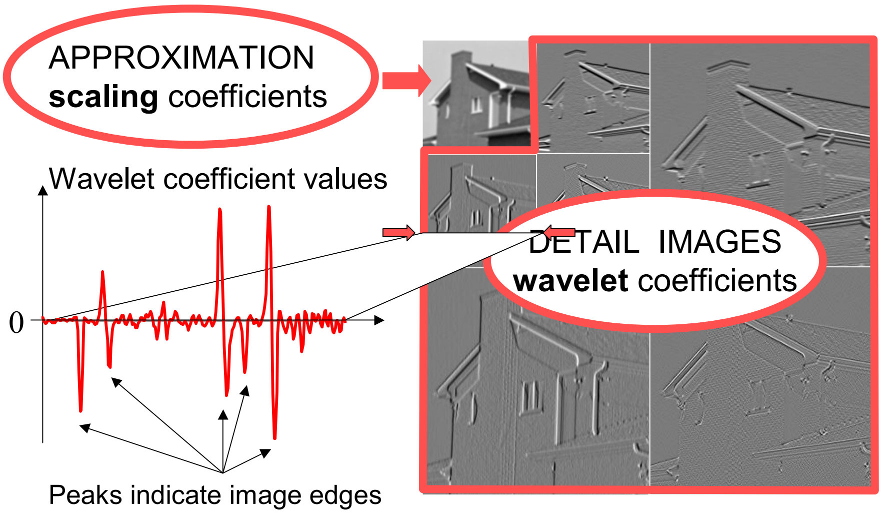


Two dimensional DWT



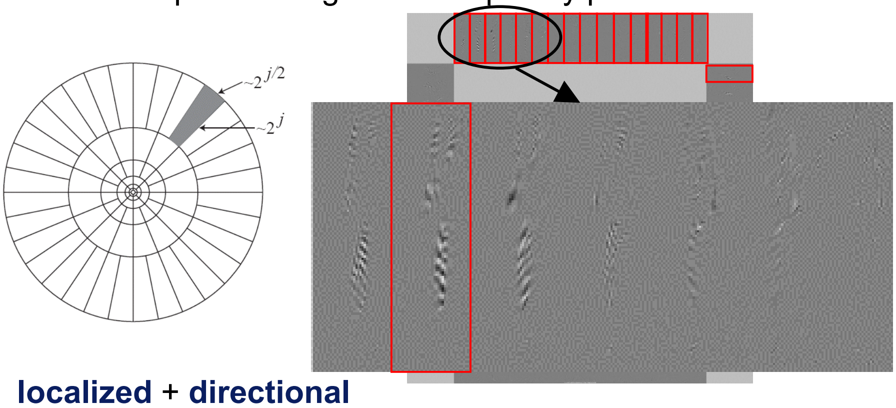


Two dimensional DWT



Curvelet transform

Curvelets: specific tiling of the frequency plane:



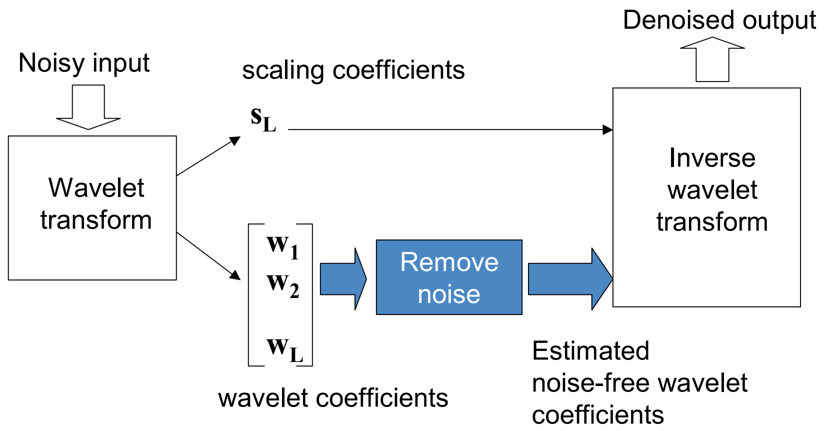
localized + directional

Excersizes

Excersize 1

Wavelet domain image denoising

Wavelet domain denosing



Noise variance estimation

- Often the value of the input noise is unknown
- Noise has to be estimated from the observed noisy signal eliminating the influence of the actual signal
- A median measurement is highly insensitive to outliers
- Meadian Absolute Deviation (MAD) estimator

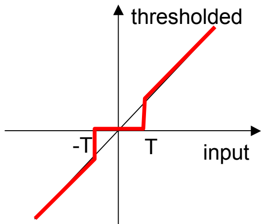
$$\hat{\sigma} = Median(|\mathbf{w}_1^{HH}|)/0.6745$$

Denoising by wavelet thresholding

$w = y + n$ w – noisy coefficient; y – noise-free coefficient; n – i.i.d. Gaussian noise

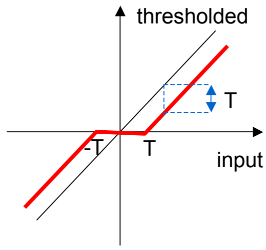
$$\hat{y}_{ht} = \begin{cases} 0, & |w| < T \\ w, & |w| \geq T \end{cases}$$

Hard thresholding "keep or kill"



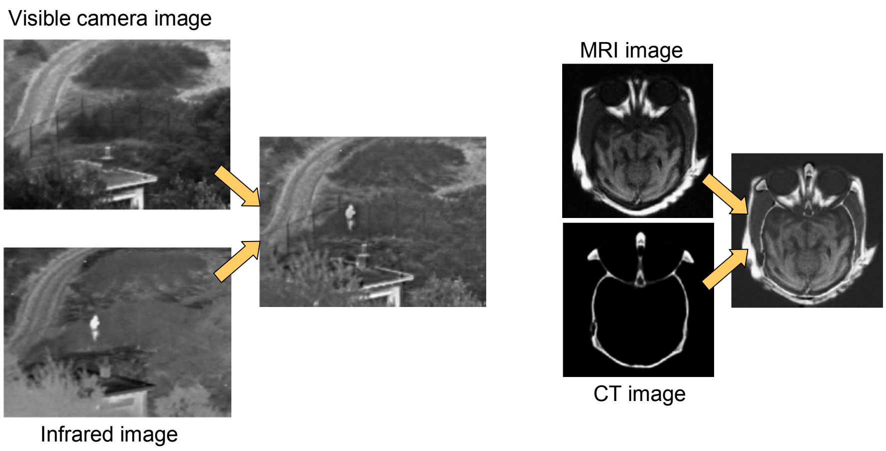
$$\hat{y}_{st} = \begin{cases} 0, & |w| < T \\ \text{sgn}(w)(|w| - T), & |w| \geq T \end{cases}$$

Soft thresholding "shrink or kill"



Excersize 2
Wavelet domain image fusion

Applications in image fusion…



… Applications in image fusion