UNIVERSITEIT GENT

FACULTY OF ENGINEERING

Sparse Coding and Multimodal Dictionary Learning in Computer Vision

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Outline

1. Sparse Coding via Dictionary Learning
   - Sparse representation
   - Some Open Problems and Challenges
2. Coupled Dictionary Learning
   - Image and Video Super-Resolution
   - Source Separation
3. Sparse Representation Classification
   - Hyperspectral Image Classification
   - Paint Loss Detection
4. Applications in Spectral Unmixing
   - Spectral Mixing
   - Sparse Unmixing
5. Summary

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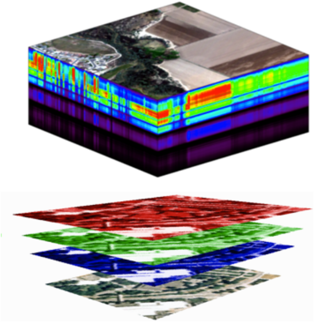
A Wealth of High-Dimensional Multimodal Data

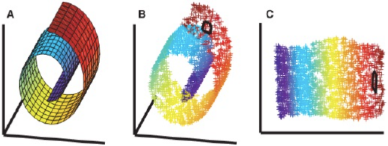


Remote sensing data (hyperspectral, visible, LiDAR,...)

Digitized paintings (infrared, X-Ray, visible)

Capturing Intrinsic Structure of the Data

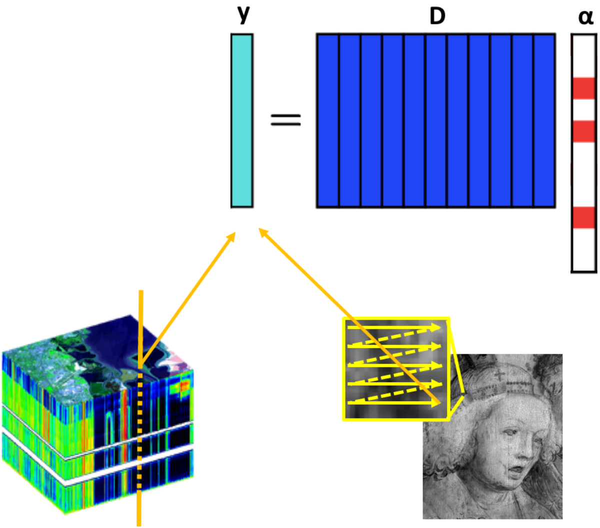
- High-dimensional data often exhibits **low-dimensional structure**
- Early models (PCA): find a linear subspace in which data resides
- Recent methods
  - Capture more complex low-dimensional structures (manifolds or unions of multiple linear subspaces)



P. V. Dinh: Review on Manifold Learning, 2009

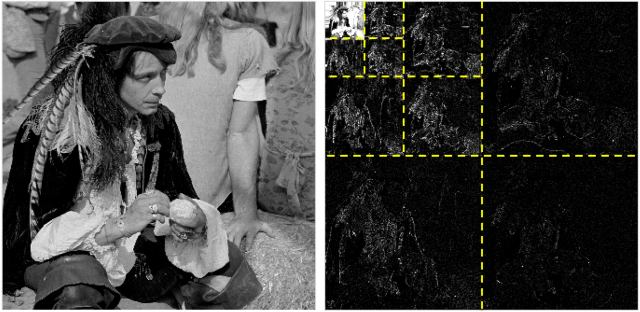
  - Data has a sufficiently **sparse representation** with respect to some basis or a dictionary

Sparse representation



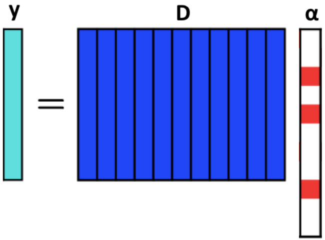
Designed vs. Learned Dictionaries

- **Designed dictionaries**: wavelets, curvelets, shearlets...
  - typically yield sparse representation of signals and images
  - advantages: generic, fast computation



- **Learned dictionaries**
  - trained on a set of representative examples
  - goal: optimally sparse representation for a given class of signals

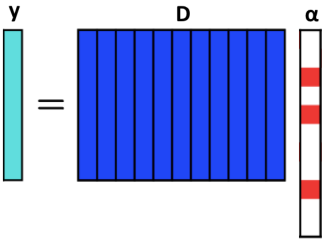
Sparse coding



$$\hat{\boldsymbol{\alpha}} = \arg\min_{\boldsymbol{\alpha}} \|\mathbf{y} - \mathbf{D}\boldsymbol{\alpha}\|_2^2 \quad \text{subject to} \quad \|\boldsymbol{\alpha}\|_0 \leq K$$

$$\hat{\boldsymbol{\alpha}} = \arg\min_{\boldsymbol{\alpha}} \|\boldsymbol{\alpha}\|_0 \quad \text{subject to} \quad \|\mathbf{y} - \mathbf{D}\boldsymbol{\alpha}\|_2^2 \leq \epsilon$$

Sparse coding



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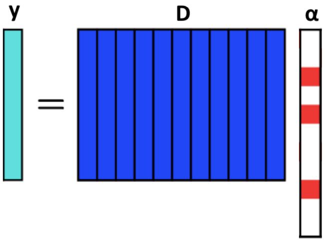
Greedy algorithms

- Matching Pursuit (MP) [Mallat and Zhang, '93]
- **OMP** [Tropp,'04], CoSaMP [Needell and Tropp, '09]
- IHT [Blumensath and Davies, 09]

Sparse coding



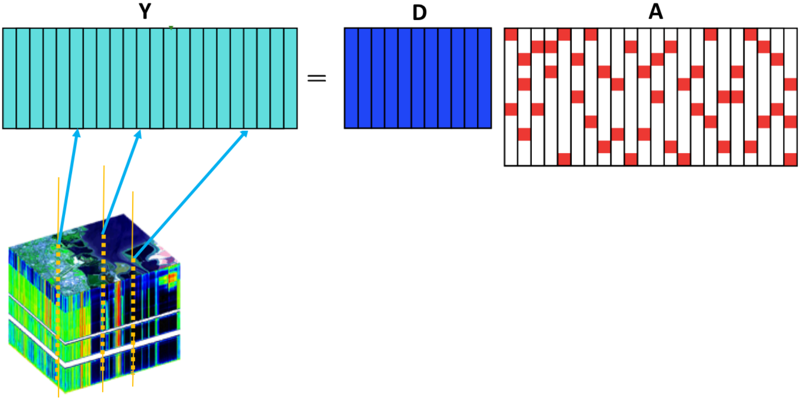
Convex relaxation:

$$\hat{\boldsymbol{\alpha}} = \arg\min_{\boldsymbol{\alpha}} \|\boldsymbol{\alpha}\|_1 \quad \text{subject to} \quad \|\mathbf{y} - \mathbf{D}\boldsymbol{\alpha}\|_2^2 \leq \epsilon$$

$$\hat{\boldsymbol{\alpha}} = \arg\min_{\boldsymbol{\alpha}} \|\mathbf{y} - \mathbf{D}\boldsymbol{\alpha}\|_2^2 + \lambda\|\boldsymbol{\alpha}\|_1$$

LASSO [Tibshirani, '96], BPDN [Chen et al, '01]

Sparse coding and dictionary learning

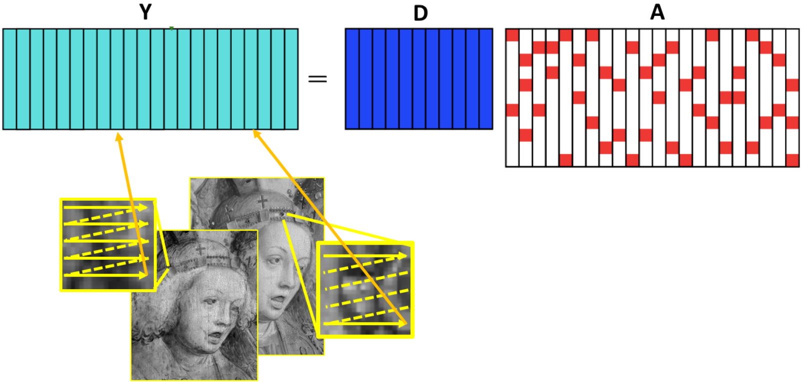


$$\{\hat{\mathbf{D}}, \hat{\mathbf{A}}\} = \arg\min_{\mathbf{D},\mathbf{A}} \left\{ \|\mathbf{Y} - \mathbf{D}\mathbf{A}\|_F^2 \right\} \quad \text{subject to} \quad \forall i, \|\boldsymbol{\alpha}_i\|_0 \leq K$$

A similar objective:

$$\{\hat{\mathbf{D}}, \hat{\mathbf{A}}\} = \arg\min_{\mathbf{D},\mathbf{A}} \sum_i \|\boldsymbol{\alpha}_i\|_0 \quad \text{subject to} \quad \|\mathbf{Y} - \mathbf{D}\mathbf{A}\|_F^2 \leq \epsilon$$

Sparse coding and dictionary learning



$$\{\hat{\mathbf{D}}, \hat{\mathbf{A}}\} = \arg\min_{\mathbf{D},\mathbf{A}} \left\{ \|\mathbf{Y} - \mathbf{D}\mathbf{A}\|_F^2 \right\} \quad \text{subject to} \quad \forall i, \|\boldsymbol{\alpha}_i\|_0 \leq K$$

A similar objective:

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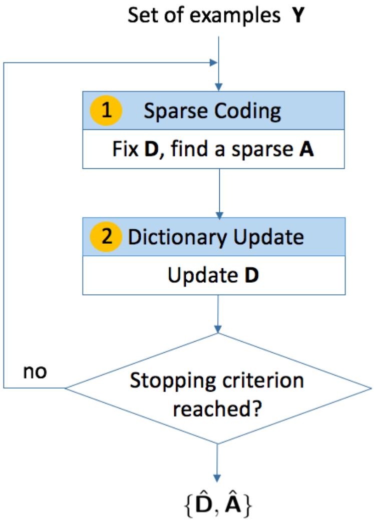
Iterate Two Steps: Sparse Coding and Dictionary Update



Dictionary update:

- method of Olshausen&Field
- MOD
- K-SVD

Sparse Coding - A Strategy Employed by V1?

[Olshausen and Field, 1997]

Idea: **maximize the likelihood** :

$$P(\mathbf{Y}|\mathbf{D}) = \prod_i P(\mathbf{y}_i|\mathbf{D}) = \prod_i \int P(\mathbf{y}_i|\boldsymbol{\alpha}, \mathbf{D}) P(\boldsymbol{\alpha}) d\boldsymbol{\alpha}$$

Approximate $P(\mathbf{y}_i|\mathbf{D})$ via extremal values. Require: $P(\boldsymbol{\alpha}) \propto e^{-\lambda|\boldsymbol{\alpha}|}$

$$\begin{aligned}\hat{\mathbf{D}} &= \arg\max_{\mathbf{D}} \sum_i \max_{\boldsymbol{\alpha}_i} \Big\{ P(\mathbf{y}_i|\boldsymbol{\alpha}_i, \mathbf{D}) P(\boldsymbol{\alpha}_i) \Big\} \\ &= \arg\min_{\mathbf{D}} \sum_i \min_{\boldsymbol{\alpha}_i} \Big\{ \|\mathbf{D}\boldsymbol{\alpha}_i - \mathbf{y}_i\|^2 + \lambda \|\boldsymbol{\alpha}\|_1 \Big\}\end{aligned}$$

Two-step iterative procedure :

1. Calculate $\{\boldsymbol{\alpha}_i\}_{i=1}^N$ using a **gradient descent** procedure
2. Update the dictionary as: $\mathbf{D}^{(k+1)} = \mathbf{D}^{(k)} - \eta \sum_{i=1}^N \Big(\mathbf{D}^{(k)} \boldsymbol{\alpha}_i - \mathbf{y}_i \Big) \boldsymbol{\alpha}_i^T$

The Method of Optimal Directions (MOD) method

[Engan et al., 1999]

Assume the sparse code for each example is known, and define the coding errors: $\mathbf{e}_i = \mathbf{y}_i - \mathbf{D}\boldsymbol{\alpha}$

Idea: **minimize the overall representation error**:

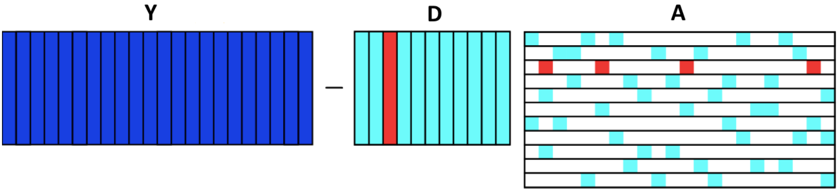
$$\|\mathbf{E}\|_F^2 = \|[\mathbf{e}_1, \ldots, \mathbf{e}_N]\|_F^2 = \|\mathbf{Y} - \mathbf{D}\mathbf{A}\|_F^2$$

Two-step iterative procedure :

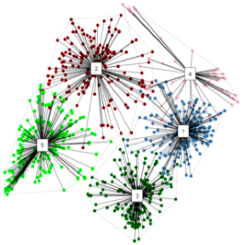
1. Calculate $\mathbf{A} = [\boldsymbol{\alpha}_1, \ldots \boldsymbol{\alpha}_N]$ using **OMP**
2. Update the dictionary as: $\mathbf{D}^{(k+1)} = \mathbf{Y}{\mathbf{A}^{(k)}}^T \left(\mathbf{A}^{(k)}{\mathbf{A}^{(k)}}^T\right)^{-1}$

K-SVD

[Aharon et al., 2006]

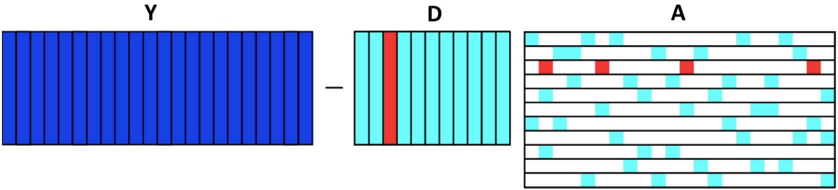


Like in K-means clustering, update one dictionary atom at a time



K-SVD (continued)

[Aharon et al., 2006]

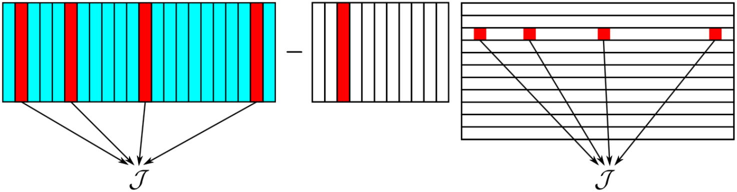


Like in K-means clustering, update one dictionary atom at a time

$$\|\mathbf{Y}-\mathbf{DA}\|_F^2 = \left\|\mathbf{Y}-\sum_j \mathbf{d}_j\boldsymbol{\alpha}_j^T\right\|_F^2 = \Bigg\| \underbrace{\left(\mathbf{Y}-\sum_{j\neq k}\mathbf{d}_j\boldsymbol{\alpha}_j^T\right)}_{\mathbf{E}_k = \text{error due to omitting } \mathbf{d}_k} - \mathbf{d}_k\boldsymbol{\alpha}_k^T \Bigg\|_F^2$$

K-SVD (continued)

[Aharon et al., 2006]



Restrict the calculation of the error $\mathbf{E}_k$ only to columns $\mathbf{y}_i$ that use the atom $\mathbf{d}_k$, i.e., only to columns

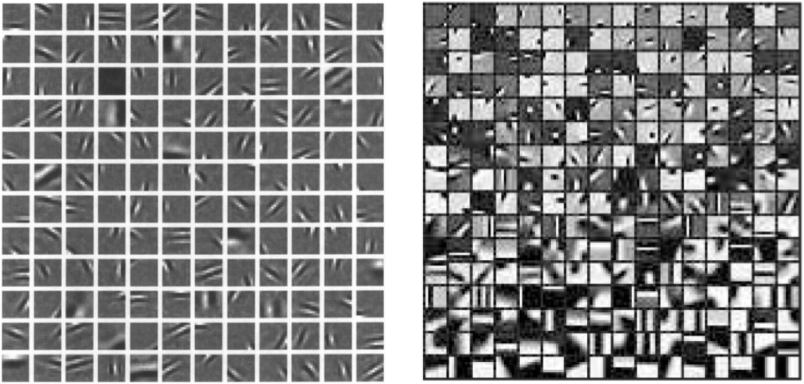
$$\mathcal{J} = \{j \mid 1 \leq j \leq K, \boldsymbol{\alpha}_k^T(j) \neq 0\}$$

Apply SVD to the restricted error

$$\mathbf{E}_k^R = \mathbf{U}\boldsymbol{\Delta}\mathbf{V}^T$$

Update $\mathbf{d}_k$ to be the first column of $\mathbf{U}$;
Update $\boldsymbol{\alpha}_k^T$ to be the first column of $\mathbf{V}$ multiplied by $\boldsymbol{\Delta}(1,1)$

Learned Dictionaries of Image Atoms - Examples



Examples of dictionaries trained by [Olshausen and Field, 1997] (left) and K-SVD [Aharon et al., 2006] (right)

Unsupervised vs. Supervised Dictionary Learning

- **Unsupervised** dictionary learning

$$\{\hat{\mathbf{D}}, \hat{\mathbf{A}}\} = \arg\min_{\mathbf{D},\mathbf{A}} \left\{ \|\mathbf{Y} - \mathbf{DA}\|_F^2 \right\} \quad \text{subject to} \quad \forall i, \|\boldsymbol{\alpha}_i\|_0 \leq K$$

  - minimizes the reconstruction error
  - inverse problems (restoration, inpainting,...)

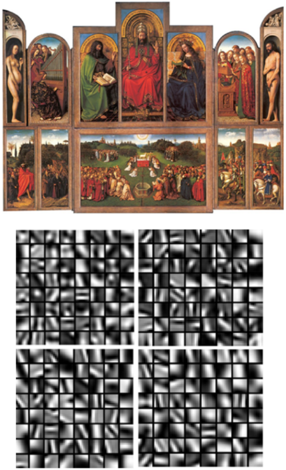
- **Supervised** (discriminative or **task-driven**)

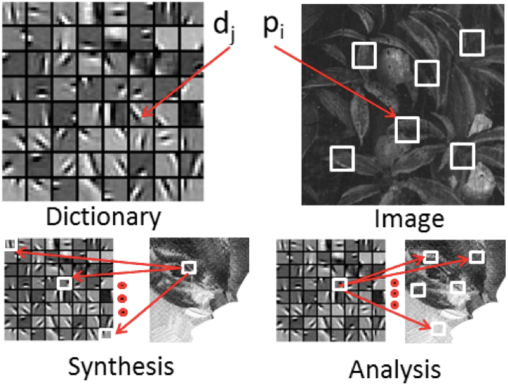
$$\{\hat{\mathbf{D}}, \underbrace{\hat{\mathbf{C}}}_{\text{class. par.}}, \hat{\mathbf{A}}\} = \arg\min_{\mathbf{D},\mathbf{C},\mathbf{A}} \left\{ \|\mathbf{Y} - \mathbf{DA}\|_F^2 + \mu \|\underbrace{\mathbf{H}}_{\text{labels}} - \mathbf{CA}\|_F^2 + \eta \|\mathbf{C}\|_F^2 \right\}$$

$$\text{subject to} \quad \forall i, \|\boldsymbol{\alpha}_i\|_0 \leq K$$

  - classification problems (**H** – label inform.; **C** – classifier parameters)

Application in Painter Style Characterization





[Hughes et al, 2009], [Latić and Pižurica, 2014]

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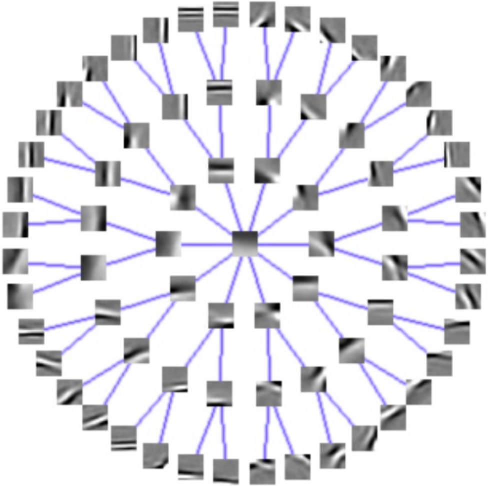
Hierarchical Dictionaries



J. Mairal, R. Jenatton, G. Obozinski and F. Bach (2011). Learning Hierarchical and Topographic Dictionaries with Structured Sparsity. SPIE Wav.&Sparsity.

Some Open Problems and Challenges

- Most of the current methods focus on training a dictionary in a single feature space or on features derived from the same imaging modality [Shen et al., 2015]. **Generalizations to multimodal and heterogeneous data** are challenging.

- Development of **efficient learning tools**, especially for multimodal dictionaries is still an open problem [Bahrampour et al., 2006].

- **Lack of structure** limits the practical applicability of learned dictionaries [Shao et al., 2014].

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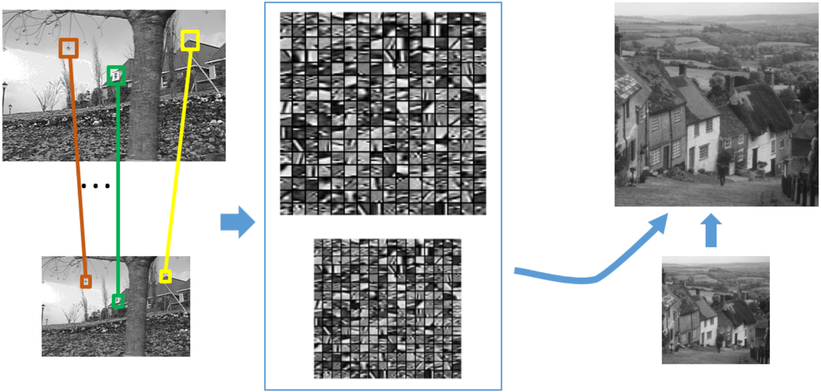
Coupled Dictionary Learning for Super-Resolution



Coupled dictionary learning

SR reconstruction

J. Yang, J. Wright, T. S. Huang, and Y. Ma (2010). Image Super-Resolution Via Sparse Representation, IEEE TIP.

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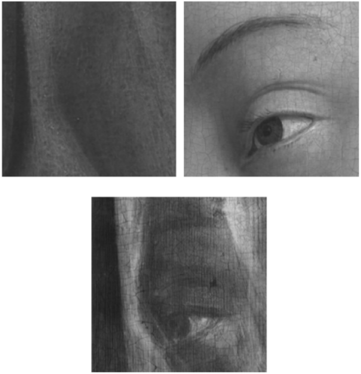
Coupled Dictionary Learning for X-Ray Source Separation

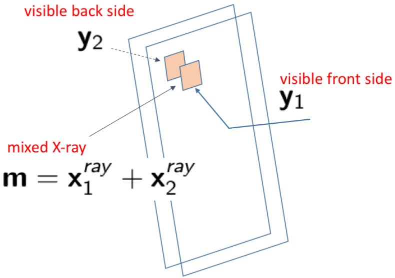
Visible parts from the front and back panel



X-ray scan

N. Deligiannis, J. Mota, B. Cornelis, M. Rodrigues and I. Daubechies (2017). Multi-Modal Dictionary Learning for Image Separation With Application in Art Investigation. IEEE TIP.

Coupled Dictionary Learning for X-Ray Source Separation



$$\mathbf{y}_1 = \boldsymbol{\Psi}^c \mathbf{z}_{1c}$$

$$\mathbf{y}_2 = \boldsymbol{\Psi}^c \mathbf{z}_{2c}$$

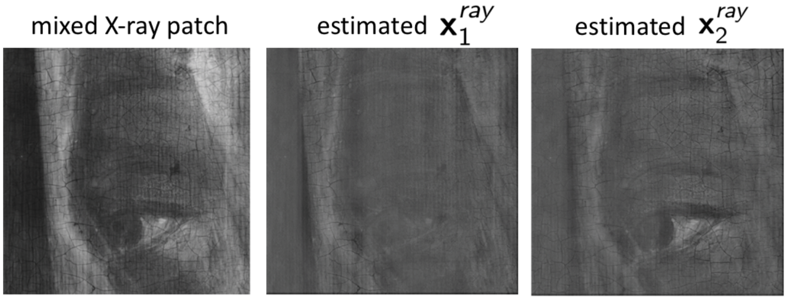
$$\mathbf{x}_1^{ray} = \boldsymbol{\Phi}^c \mathbf{z}_{1c} + \boldsymbol{\Phi}\mathbf{v}$$

$$\mathbf{x}_2^{ray} = \boldsymbol{\Phi}^c \mathbf{z}_{2c} + \boldsymbol{\Phi}\mathbf{v}$$

$$\begin{aligned} \min \; & \left\{ \|\mathbf{z}_{1c}\|_1 + \|\mathbf{z}_{2c}\|_1 + 2\|\mathbf{v}\|_1 \right\} \\ \text{subject to} \; & \mathbf{m} = \boldsymbol{\Phi}^c \mathbf{z}_{1c} + \boldsymbol{\Phi}^c \mathbf{z}_{2c} + \boldsymbol{\Phi}\mathbf{v} \\ & \mathbf{y}_1 = \boldsymbol{\Psi}^c \mathbf{z}_{1c} \\ & \mathbf{y}_2 = \boldsymbol{\Psi}^c \mathbf{z}_{2c} \end{aligned}$$

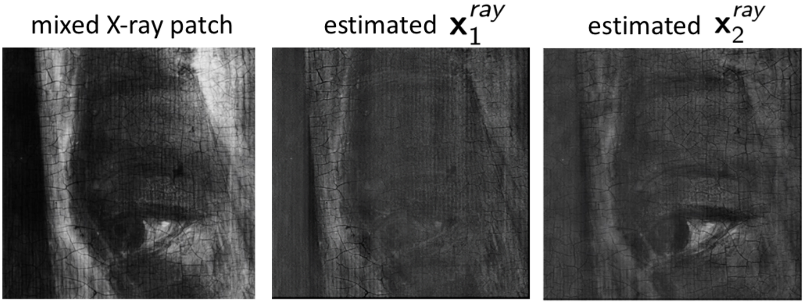
[Deligiannis et al, 2017]

Coupled Dictionary Learning for X-Ray Source Separation



[Deligiannis et al, 2017]

Coupled Dictionary Learning for X-Ray Source Separation



(contrast enhanced)

[Deligiannis et al, 2017]

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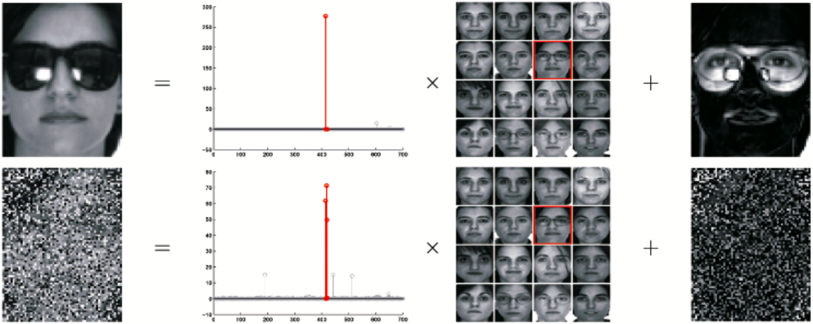
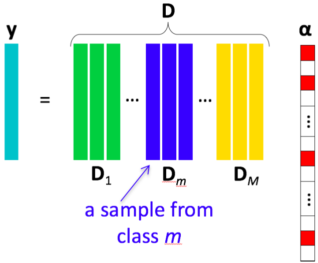
Sparse Representation Classification - SRC



Wright, J., Yang, A. Y., Ganesh, A., Sastry, S. S., and Ma, Y. (2009). Robust face recognition via sparse representation. IEEE PAMI.

Sparse Representation Classification - SRC



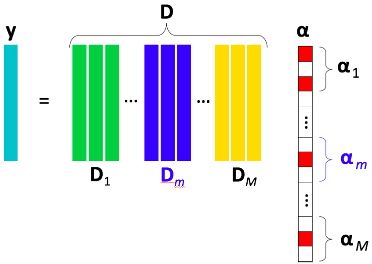
$$\hat{\boldsymbol{\alpha}} = \arg\min_{\boldsymbol{\alpha}} \|\mathbf{y} - \mathbf{D}\boldsymbol{\alpha}\|_2^2 \quad s.t. \quad \|\boldsymbol{\alpha}\|_0 \leq K$$

Let $\delta_m(\boldsymbol{\alpha})$ denote a vector whose all entries are set to zero except those associated with class m and define a class-specific residual $r_m(\mathbf{y})$

$$r_m(\mathbf{y}) = \|\mathbf{y} - \mathbf{D}\delta_m(\hat{\boldsymbol{\alpha}})\|_2, \quad m = 1, ..., M$$

and the sample class as: $identity(\mathbf{y}) = \arg\min_{m=1,\ldots,M} r_m(\mathbf{y})$

Sparse Representation Classification - SRC



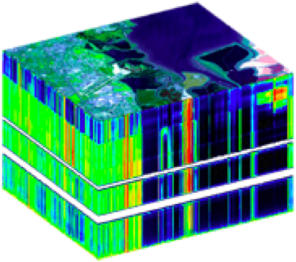
Equivalently,

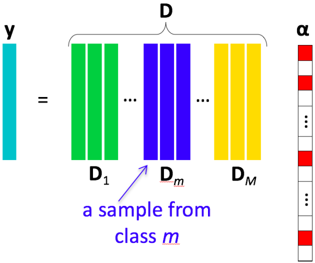
$$r_m(\mathbf{y}) = \|\mathbf{y} - \mathbf{D}_m \hat{\alpha}_m\|_2, \qquad m = 1, ..., M$$

$$identity(\mathbf{y}) = \arg\min_{m=1,\dots,M} r_m(\mathbf{y})$$

SRC in Hyperspectral Image Classification

[Chen et al., 2011]





$$\hat{\alpha} = \arg\min_{\alpha} \|\mathbf{y} - \mathbf{D}\alpha\|_2^2 \quad \text{subject to } \|\alpha\|_0 \leq K$$

$$r_m(\mathbf{y}) = \|\mathbf{y} - \mathbf{D}_m\hat{\alpha}_m\|_2, \qquad m = 1, \ldots, M$$

$$class(\mathbf{y}) = \arg\min_{m=1,\ldots,M} r_m(\mathbf{y})$$

Joint Sparsity Model

Collect pixels from a small neighbourhood $\mathcal{N}_\epsilon$ into $\mathbf{Y} = [\mathbf{y}_1, ..., \mathbf{y}_T] \in \mathbb{R}^{B \times T}$

$$\mathbf{Y} = \underbrace{[\mathbf{y}_1 \ \ldots \ \mathbf{y}_T]}_{\text{pixels from } \mathcal{N}_\epsilon} = [\mathbf{D}\boldsymbol{\alpha}_1 \ \ldots \ \mathbf{D}\boldsymbol{\alpha}_T] = \mathbf{D}\underbrace{[\boldsymbol{\alpha}_1 \ \ldots \ \boldsymbol{\alpha}_T]}_{\mathbf{A}} = \mathbf{DA}$$

Sparse codes $\{\boldsymbol{\alpha}_t\}_{t=1}^T$ share the same support $\implies$ **A** is sparse with only K non-zero rows, i.e., **A** is row sparse.

JSRC method [Chen et al., 2011]:

$$\hat{\mathbf{A}} = \arg\min_{\mathbf{A}} \|\mathbf{Y} - \mathbf{DA}\|_F^2 \ \text{ subject to } \|\mathbf{A}\|_{row,0} \leq K$$

$$r_m(\mathbf{Y}) = \|\mathbf{Y} - \mathbf{D}_m\hat{\mathbf{A}}_m\|_F, \qquad m = 1, ..., M$$

$$class(\mathbf{y}_{central}) = \arg\min_{m=1,\ldots,M} r_m(\mathbf{Y})$$

Robust SRC for Hyperspectral Image Classification

$$\mathbf{Y} = \underbrace{\mathbf{X}}_{\text{ideal image}} + \underbrace{\mathbf{N}}_{\text{Gaussian noise}} + \underbrace{\mathbf{S}}_{\text{sparse noise}}$$


Examples of stripe noise and mixed noise in a real hyperspectral image.

Huang, S., Zhang, H., Liao, W., and Pižurica, A. (2017). Robust joint sparsity model for hyperspectral image classification. In IEEE ICIP 2017

Robust SRC for Hyperspectral Image Classification

$$\mathbf{Y} = \underbrace{\mathbf{X}}_{\text{ideal image}} + \underbrace{\mathbf{N}}_{\text{Gaussian noise}} + \underbrace{\mathbf{S}}_{\text{sparse noise}}$$

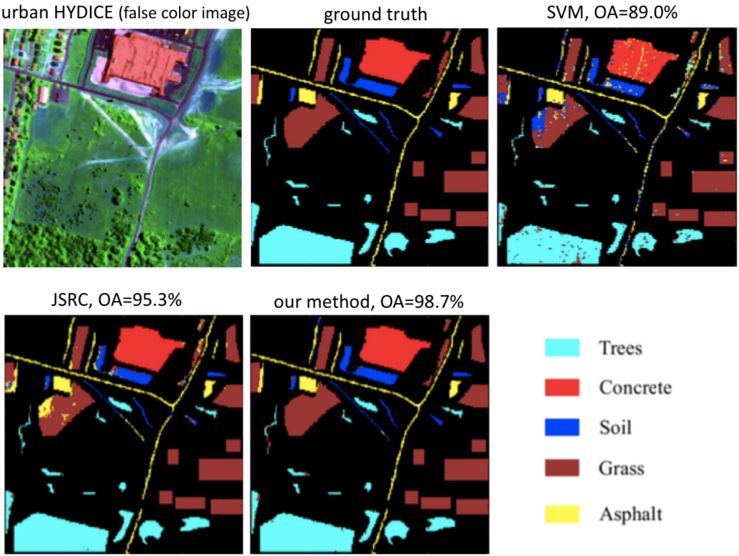
$$\{\hat{\mathbf{A}}, \hat{\mathbf{S}}\} = \arg\min_{\mathbf{A},\mathbf{S}} \|\mathbf{Y} - \mathbf{DA} - \mathbf{S}\|_F^2 + \lambda\|\mathbf{S}\|_1 \quad \text{subject to} \quad \|\mathbf{A}\|_{row,0} \leq K$$

$$r_m(\mathbf{Y}) = \|\mathbf{Y} - \mathbf{D}_m\hat{\mathbf{A}}_m - \hat{\mathbf{S}}\|_F, \qquad m = 1, ..., M$$

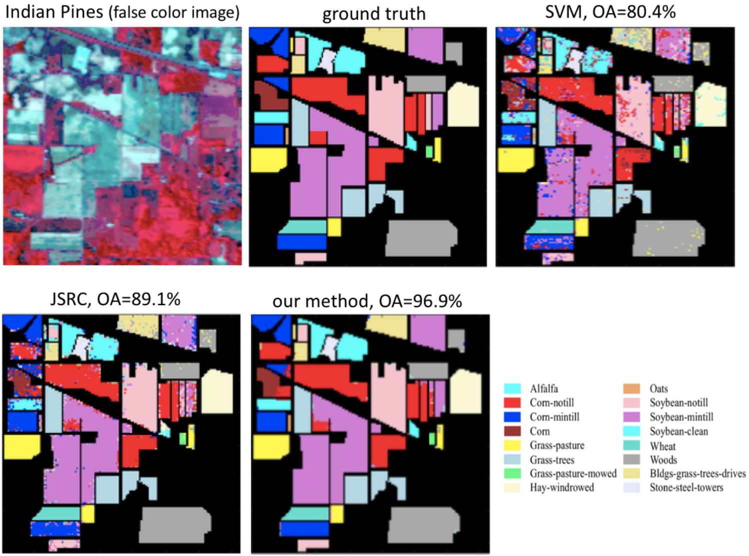
$$class(\mathbf{y}_{central}) = \arg\min_{m=1,\ldots,M} r_m(\mathbf{Y})$$

Huang, S., Zhang, H., Liao, W., and Pižurica, A. (2017). Robust joint sparsity model for hyperspectral image classification. In IEEE ICIP 2017

Robust SRC for Hyperspectral Image Classification



Robust SRC for Hyperspectral Image Classification



Outline

1. Sparse Coding via Dictionary Learning
   - Sparse representation
   - Some Open Problems and Challenges
2. Coupled Dictionary Learning
   - Image and Video Super-Resolution
   - Source Separation
3. Sparse Representation Classification
   - Hyperspectral Image Classification
   - Paint Loss Detection
4. Applications in Spectral Unmixing
   - Spectral Mixing
   - Sparse Unmixing
5. Summary

The *Ghent Altarpiece*

Image copyright: Ghent, Kathedrale Kerkfabriek, Lukasweb

Also known as *Adoration of the Mystic Lamb*, Hubert and Jan Van Eyck, completed in 1432.

Ghent Altarpiece - Current Restoration Campaign

Art and design

theguardian

Restored and ravishing: the magnificent Ghent Altarpiece gives up its centuries-old mysteries

It is one of the most influential, and most stolen, works ever. But for centuries, the origins of the Ghent Altarpiece have been shrouded in mystery. Now a restoration is revealing the truth about this masterpiece

Noah Charney

Wednesday 12 October 2016

Ghent Altarpiece - Current Restoration Campaign

SCIENCE

The New York Times

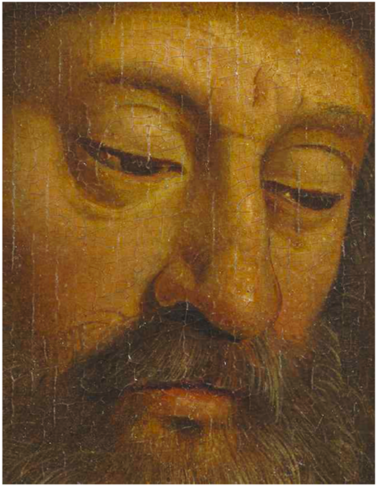
A Master Work, the Ghent Altarpiece, Reawakens Stroke by Stroke

By MILAN SCHREUER DEC. 19, 2016

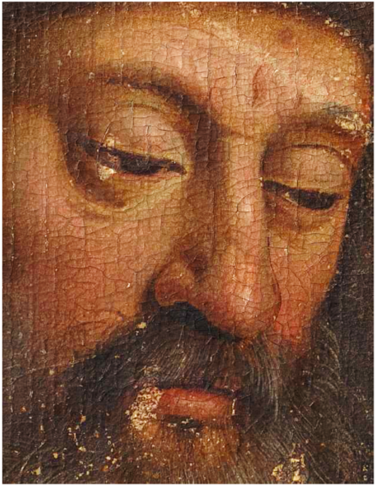
Ghent Altarpiece - Current Restoration Campaign

Image copyright: Ghent, Kathedrale Kerkfabriek, Lukasweb

Ghent Altarpiece - Current Restoration Campaign



before cleaning



after cleaning

Image copyright: Ghent, Kathedrale Kerkfabriek, Lukasweb

Problem of Paint Loss Detection

Paint loss detection crucial for

- documenting purpose
- virtual restoration
- decision making in the actual restoration process

Currently done manually:

- labor intensive
- only rough indication
- prone to errors

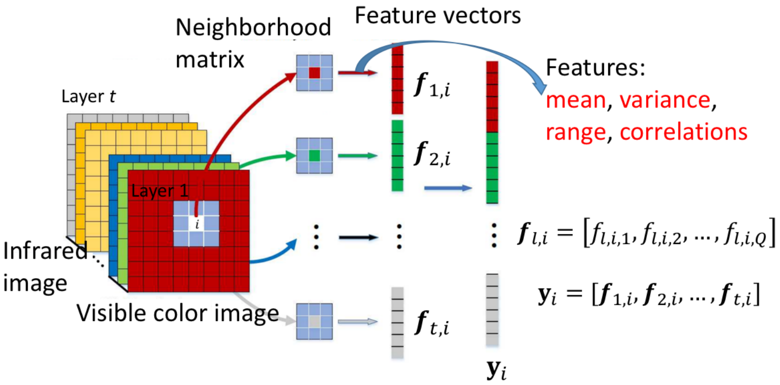
Multimodal Data

Infrared image before cleaning

Visible image before cleaning

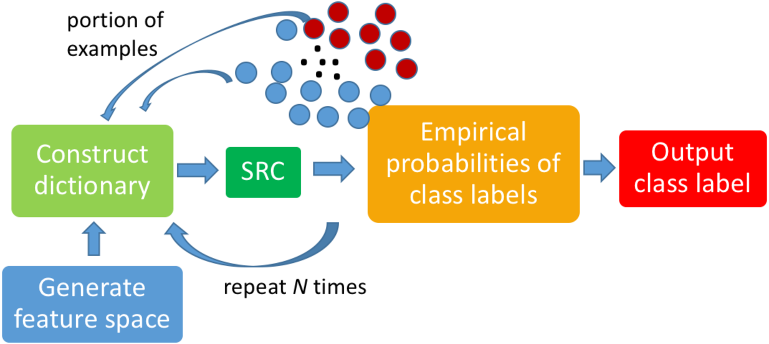
Visible image after cleaning

Features for SRC



S. Huang, W. Liao, H. Zhang, and A. Pižurica (2016). Paint Loss Detection in Old Paintings by Sparse Representation Classification. iTWIST.

SRC-based Paint Loss Detection Method



N_j^m - number of trials in which $\mathbf{y}_j$ was labelled as class m;
$m \in \{PaintLoss, Other\}$

$$class(\mathbf{y}_j) = \arg\max_m p_j(m) = \underbrace{\arg\max_m \,(N_j^m/N)}_{\text{empirical prob. of class } m}$$

S. Huang, W. Liao, H. Zhang, and A. Pižurica (2016). Paint Loss Detection in Old Paintings by Sparse Representation Classification. iTWIST.

Paint Loss Detection Results

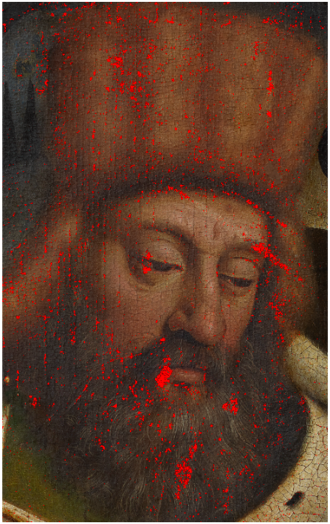


Image copyright: Ghent, Kathedrale Kerkfabriek, Lukasweb

Virtual Restoration

The result of automatic paint loss detection followed by inpainting using the method of [Ružić and Pižurica, 2015].

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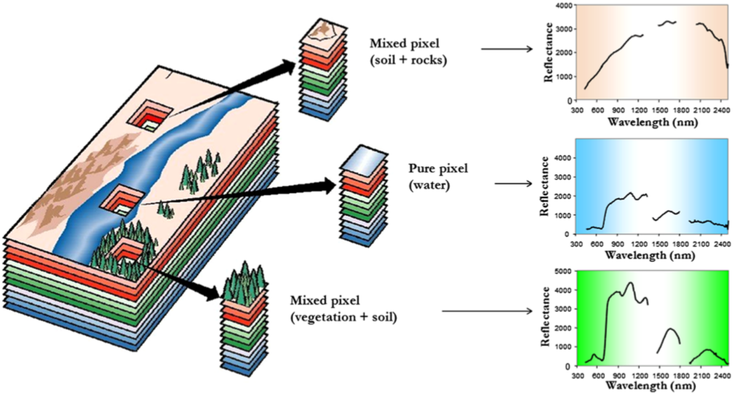
Spectral Mixing



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Sparse Unmixing

Ideal hyperspectral image reordered as a matrix $\mathbf{X} \in \mathbb{R}^{B \times MN}$
Linear mixing model:

$$\mathbf{X} = \mathbf{E}\mathbf{A}$$

$\mathbf{E} \in \mathbb{R}^{B \times K}$ – library of **endmembers**; $\mathbf{A} \in \mathbb{R}^{K \times MN}$ – **abundance**

$$\mathbf{Y} = \underbrace{\mathbf{E}}_{\text{library}} \underbrace{\mathbf{A}}_{\text{abundance}} + \underbrace{\mathbf{N}}_{\text{Gaussian noise}} + \underbrace{\mathbf{S}}_{\text{sparse noise}}$$

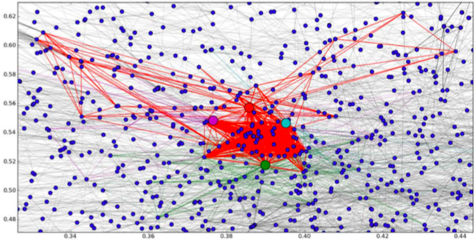
The approach of [Aggarval et al, 2016]:

$$\min_{\mathbf{A},\mathbf{S}} \|\mathbf{Y} - \mathbf{E}\mathbf{A} - \mathbf{S}\|_F^2 + \lambda_1 \|\mathbf{A}\|_{2,1} + \lambda_2 \|\mathbf{S}\|_1$$

Many similar variants exist, also making use of low-rank assumption:

$$\min_{\mathbf{A}} \operatorname{rank}\{\mathbf{A}\} \quad \text{subject to} \quad \|\mathbf{Y} - \mathbf{E}\mathbf{A} - \mathbf{S}\|_F^2 \leq \epsilon$$

Related work



Anomalies in ArXiv collaboration network (General Relativity co-authors). Georgios B. Giannakis, 2017

- Model **anomalies** in a network with a sparse outlier matrix

$$\mathbf{Y} = \mathbf{X} + \mathbf{N} + \underbrace{\mathbf{S}}_{\text{outliers}}$$

- Low-rank plus sparsity-promoting estimator

$$\min_{\mathbf{X},\mathbf{S}} \|\mathcal{P}_\Omega(\mathbf{Y} - \mathbf{X} - \mathbf{S})\|_F^2 + \lambda_1\|\mathbf{X}\|_\star + \lambda_2\|\mathbf{S}\|_1$$

M. Mardani, G. Mateos, and G. B. Giannakis (2013). Recovery of low rank plus compressed sparse matrices with application to unveiling traffic anomalies. IEEE Trans. Info. Theory.

Summary

- Many applications in various inverse problems and detection/classification tasks
- Related approaches include Sparse Representation Classification where the dictionary is constructed from the training samples

- Outlook
  - Efficient methods for multimodal dictionary learning
  - Incorporating efficiently structure and feature hierarchies

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